A monetary theory of endogenous economic growth

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Abstract

This paper presents a model where credit money is the engine of endogenous growth, because the claims for interest on debt generate liabilities that need to be matched by higher assets and income even in the steady state. The argument is based on the utility of money as a means of payment and not as a means of exchange. Interest rates are exogenously determined by liquidity preference which is a function of uncertainty, risk aversion and monetary policy. Risk aversion reflects the curvature of the utility function of money, so that that liquidity preference and interest rates are a function of money supply (monetary policy).

Borrowing and lending are determined by liquidity preference today and affect the distribution of income tomorrow. Equilibrium of saving and investment is accomplished by adjustment in income so that the saving rate is stationary in the long run. The model hinges on the solvency constraint which ensures that growing debt liabilities are serviced by higher income. In the steady state equilibrium, the growth rate must equal the interest rate and it is then determined by technological progress and population growth. Money supply must then also grow at the compound interest rate.

The paper concludes with empirical evidence from Great Britain with data over 182 years. Evidence from a non-linear autoregressive distributed lag (NARDL) model is consistent with the theoretical claim that exogenous interest rates are a driver of economic growth.

[1] London School of Economics and Chulalongkorn University, Bangkok. I am grateful to Charles Goodhart for challenging me to write this paper, and for critical feedback from Peter Bofinger, Marcos Farag and Yasmine Abdelfattah.

Money is the most important economic institution, but it plays hardly any role in theories of economic growth. Since the Conference on Money and Economic Growth with papers published by the *Journal of Money, Credit and Banking* in 1968, no major colloquium on the theme has taken place. Maybe the high inflation of the 1970s which fuelled the neoclassical and monetarist counterrevolution has crowded out the Keynes-Wicksell approach to monetary growth.[2] Nevertheless, in the following decades progress was made by studying the channels through which financial markets and institutions affect economic development. They demonstrated a strong positive link between the functioning of the financial system and long-run economic growth.[3] The focus shifted to institutional features of the financial system away from monetary policy. Levine (1997, 721) concluded that "we do not have a sufficient understanding of long-run economic growth until we understand the evolution and functioning of financial systems". This conclusion had an important corollary: "although financial panics and recessions are critical issues, the finance-growth link goes beyond the relationship between finance and shorter-term fluctuations". The recent period of unconventional monetary policy with negative or close to zero interest rates has revealed how closely financial stability is linked to monetary policy. This warrants a new reflection on the possible link between money and long-term economic growth.

In the old Keynes-Wicksell approach, money had sharp short-term effects, but in the neoclassical synthesis it was considered neutral because a change in the level of nominal money did not affect real variables; when money was "super-neutral", a change in the growth rate of money did not affect the rate of economic growth. However, money was non-neutral when the impact of inflation on real interest rates affected economic growth.[4] In the monetarist interpretation, the growth of money supply was the driver for inflation, but how the money supply grows was hardly explained (Stein 1969). Over the last half century our understanding of the transmission mechanisms of monetary policy has improved and it is generally understood that interest rates are the key variable. Hence, a model for explaining economic growth and money must give a prominent role to interest rates.

I suggest a model that cuts through the traditional Keynes versus neoclassics controversy. I will argue that in the steady state with constant capital stock, interest rates determine the long-run growth. In line with neoclassics, the rate of capital formation is always equal to planned savings, i.e., output less planned consumption.[5] This assumes away Keynesian inflation by excess demand, which is appropriate when we consider economic growth in the long run. Money is not supplied by helicopters, but by the central bank that manages money supply through interest rates. Assuming price stability allows us to analyse the demand for money in nominal terms. This is important for understanding liquidity preference, and the role of interest rates for long-run growth. The exogenous variable in the model is liquidity preference which is determined by uncertainty, risk aversion and the utility function of money. In this perspective, I maintain the Keynesian separation of money demand for transaction and precautionary motives. However, while I set up the logic of the model in nominal terms, the empirical estimates in the last section are made in real terms to catch the effect of inflation.

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[2] On the conference, see (Stein 1969); (Fischer 1972) for the two schools' interpretation of money and growth, and (Meltzer 1969) for a critique of Keynes-Wicksell.

[3] For a review see (Levine 1997) (Levine 2005).

[4] (Orphanides and Solow 1990) give an overview.

[5] This applies to a closed economy. I assume with Keynes that the adjustment will work through short run changes in income and not through the interest rate and changes in income are affected by real exchange rates.

As Tobin has pointed out, a general equilibrium approach to monetary theory needs to define one variable as independent to which the others adjust through changes in asset prices, quantities, and income.[6] In this model, the Keynesian liquidity premium, which is a component of the relevant interest rate, plays this exogenous role. In non-monetary models the interest rate is set by the marginal productivity of capital; in monetary models, the rate of time preference is a parameter in an impatient individual's utility function which captures the trade-off between consumption today and consumption in the future. However, this assumption of impatience is an *ad hoc* device that has no further economic foundation.

Neoclassical models usually treat money as "outside money", i.e., an asset for the private economy unmatched by private liabilities.[7] This money is issued by the government to finance the budget deficit and the public liability is repaid through the inflation tax. In that case, money is a source of distortions, but not a driver of growth. Output growth is driven by the accumulation of capital and technological progress, which is the endogenous outcome of profit maximising firms. Money simply serves as a means of exchange and has no other utility. Monetary policy can affect real variables through inflation which determines the yield on real balances relative to the yield of real capital. However, in the steady state, capital accumulation stops, and the model does not explain how economic growth can grow exponentially in the steady state.

I seek to overcome the limitations of the neoclassical approach by a model where an exogenously set interest rate generates *exponential growth of liabilities* which requires a *commensurate growth of assets*, and that generates income growth. To clarify the basic logic of the argument, I focus on "inside money", which is the liability of the banking system in the consolidated balance sheet of central and commercial banks. Money supply is endogenous to the interest rate.[8] The exogenous interest rate therefore becomes the engine of economic growth. Contrary to neoclassical models, this result even holds under conditions of price stability.

This paper will first explain the utility of money and explain the interest rate as an exogenous variable derived from liquidity preference in an uncertain world. In the second section, I look at the dynamics of growth in a monetary economy and the intertemporal effects of saving, lending, and borrowing. It establishes the warranted growth rate which satisfies the solvency constraint and shows that in the steady state this condition requires technological progress. Section three will study evidence from 182 years of growth in England. It confirms the standard theory of increasing growth by stimulating capital accumulation through lower interest rates, but it also confirms that higher interest rate engender growth presumably through increasing productivity in the steady state.

1. Liquidity preference and the utility of money

The building blocks of the model are as follows. Output consists of one produced good, which can be consumed directly or accumulated as capital. Money is the liability of the banking system (central bank and commercial banks). We assume a closed economy, so that total wealth represents the value of physical capital. The stock of capital is *in the possession* of firms. Households are the owners of total

[6] (Tobin 1969). See also (Tobin 1961): "The independent interest rate in an aggregative system should be enumerated. An independent rate is one that is not tied to another yield by an invariant relationship determined outside the system; e.g., by a constant risk differential."
[7] Typical models are Tobin (1965) and Solow (2000, Chap. 4).
[8] This is the modern view of money creation, see (McLeay, Michael; Amar Radia; Ryland Thomas 2014). On endogenous money see (Moore 1988).

wealth and their *assets are claims to possession*. Liabilities are the obligation to give access to capital or money. The facility of access to the possession defines the degree of liquidity of asset claims.[9] These financial claims are fixed by contractual agreements. We distinguish two financial assets: money gives immediate access to resources because it has purchasing power; bonds give immediate access to money for borrowers, but they are claims on money in the future for lenders. Bonds are issued by firms which borrow money from households or banks. Equity is the residual value of capital after deducting the obligation to repay borrowed money. Money consists of units of account (currency units) that are created when the banking system is making loans (i.e., "buying" bonds) that are credited as deposits. They also hold reserves as claims against the central bank. These claims, which are the liability of the central bank, are "money proper" (high-powered money) insofar the currency units issued by the central bank serve as the legal instrument to discharge debt contracts irrevocably and with certainty. The government only comes into the picture in the form of setting up the central bank and making the central bank's liabilities legal tender. There is no public debt because the government's budget is and has always been balanced.

1.1. The economy's sectoral balance sheet

Table 1 describes the sectoral balance sheet (sometimes also called Capital account) of the economy's flow of funds account.[10] Changes in the sectoral balance sheet are recorded in the Generation of income account.[11] The total wealth of households (net worth) consists of claims on physical capital (K) at current prices represented by 200 currency units. These wealth-claims are held in the form of money, bonds, and equity. Firms hold no money balances because they immediately distribute them as income to households in the form of wages and profits. Equity is exclusively owned by households. Bonds are issued by firms and held by households and the banking system. Money, as the integrated liability of the banking system,[12] is created (supplied) when banks purchase bonds and its stock is reduced when the bonds are redeemed or sold to households. However, when banks are selling bonds to the central bank, they swap less-liquid assets against liquidity par excellence (high-powered money).

[9] Demsetz described liquidity as the "advantage of immediacy". See (Demsetz 1968) and (Klein and Selgin 2000, 218).

[10] Adapted from (Lagos 2006). See also (Tobin 1969).

[11] For the description of the framework in the System of National Accounts (SNA) see (Shrestha, Manick; Mink, Reimund; Fassler, Segismundo 2012)

[12] To simplify things, all money can be envisaged as deposits in commercial banks which are created when banks make loans to firms against the bonds issued. However, bank reserves which are the liability of the central bank are liquidity par excellence.

Table 1. Financial balance sheet of the economy

Households

Assets		*Liabilities*	
Money (M)	15		
Bonds (B)	10		
Equity (E)	175	NW	200

Firms

Assets		*Liabilities*	
Money (M)	-	Bonds (B)	25
Capital (K)	200	Equity (E)	175

Commercial banks

Assets		*Liabilities*	
Bonds (B)	10	Money	15
Reserves (R)	5		

Central bank

Assets		*Liabilities*	
Bonds (B)	5	Reserves (R)	5

The portfolio allocation of household assets maximises households' utility. It is customary to model portfolio shares as functions of the risks and returns of the three asset classes. Modern Portfolio Theory minimize levels of risk for given expected returns by focussing on the mean and variances (Markowitz 1952). This has come under criticism for assuming unrealistically quadratic utility functions and normally distributed asset returns. Such theories consider only the first two moments (mean and variance) in the portfolio selection model, while in the newer Expected Utility Theory, an optimal portfolio minimizes the risk for given levels of return and skewness, therefore taking into account the first three moments of the utility function. We shall see that the third moment of the utility function of money will define the conditions under which monetary policy is able to influence interest rates.

It is often claimed that the return on money is equal or close to zero. However, while there is no return on holding money in terms of money, there is a return in terms of utility, and this defines the liquidity premium as a non-pecuniary return. But this requires a utility function for money that is broader than the neoclassical means of exchange function which assumes that consumers care only about consumption and do not derive utility from money. Keynes saw liquidity as an additional argument in the utility function of money. "Why, he asked, should anyone outside a lunatic asylum wish to use money as a store of wealth? Because, partly unreasonable and partly on instinctive grounds, our desire to hold money as a store of wealth is a barometer of the degree of our distrust of our own calculations and conventions concerning the future. (…) The possession of actual money lulls our disquietude; and the premium which we require to make us part with money is the measure of the degree of our disquietude. The significance of this characteristic of money has usually been overlooked."[13] Thus, money has utility which is derived from our preference to lull our disquietude.

1.2. The utility of money

The utility of money reflects both, the purchasing power for buying useful goods, and the preference for liquidity that provides security in an uncertain world. Money gives cash holders the power of acquiring goods and assets and making payments *when needed* – and that is uncertain. The reason for holding liquid cash is that it protects against uncertainty.[14] The purchasing power of money is usually

[13] (Keynes, The General Theory and After. Part II. Defence and Development 1973, 116)
[14] This stands in the Keynesian tradition. Certainty is the highest degree of rational belief that events will turn out the way they are expected (Keynes 1973 [1921], 10; 137). When there is "distrust of our own calculations

described by an *indirect utility function* for money which describes consumers' maximal attainable utility when faced with a vector of goods prices and for given amounts of income. We shall see that this applies to transaction money in commercial contracts when the buyer parts with money and receives the utility of goods and services in return. However, in financial contracts the lender parts with money only if her preference for the security of holding money is compensated by paying a premium that covers the utility that holding an amount of money would yield. I interpret liquidity preference as the fundamental psychological condition of seeking safety in an environment of uncertainty. The liquidity premium is the price of certainty. The utility of money, from which its value is derived, depends therefore not only on its purchasing power (price stability) but also on the services it provides as a protection against uncertainty.

The full utility of money becomes apparent when we look at its function as a means of payment. As such money has purchasing power because it discharges debt contracts.[15] In a monetary economy, a purchase implies that the buyer promises to give a specified number of monetary units of account at a specified date in consideration of goods and services. This promise generates a contractual liability. The transfer of the currency units will *discharge the liability with certainty*. The utility of money arises precisely from this certainty. There is a set of conditions which must be met for money to fulfil this function, such as scarcity, stable value, etc., that I will not discuss here. The point is that when they are met, money has liquidity value.

Debt contracts are promises of payments - on the spot or in the future. As a means of payment, money discharges two kinds of contractual liabilities: commercial liabilities, which result from purchases of commodities and do not carry interest obligations, and financial liabilities created by loan contracts that carry interest. Holding money as means of payment therefore serves two purposes. First, people have a *transaction motive* when they hold money to buy commodities. Second, by holding money in cash instead of spending it, people protect themselves against the disquietude generated by uncertainties. This is the precautionary or *liquidity motive*.[16] Both motives together determine the utility and demand for money.

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and conventions concerning the future" (see footnote 13), we have fundamental uncertainty, which is a subjective category, contrary to risk which has epistemic objectivity. For the distinction of risk and uncertainty, see (Knight 1921). Nevertheless, we can model (expected) uncertainty as the subjectively perceived probability distribution around an expected value.

[15] In philosophical terms purchasing power is deontic power, see (Searle 2010). It can be *measured* by the inverse of the price level, but money's *purchasing power derives from the fact that it is accepted* as a means of payment.

[16] Keynes has distinguished between the transaction, precaution, and speculative motives for holding money. This can be described (Bofinger, Reischle und Schächter 1996) as an aggregative money demand function like

$$M^{dem} = PL(y,r) = M^{T}\left(\overset{+}{\bar{Y}}\right) + M^{Prec}\left(\overset{+}{\bar{Y}}\right) + M^{spec}(\overset{-}{\bar{r}})$$

Where the plus and minus signs indicate the sign of the first derivative. Because the precautionary motive is about converting money into real assets in the context of uncertainty, it is often fused with the transaction motive, while the speculative motive of buying financial assets is inversely related to the expectation of future interest rate and capital gains. This model is appropriate for a modern economy with efficient financial markets, but in their absence, precautionary money follows a logic that is like speculative money.

A given amount of money will provide utility by satisfying both motives generating demand for transaction money and for precautionary money. Transaction money is spent on output and therefore determines income, but it is constrained by total money supply and the precautionary balances held as protection against uncertainty.

(1) $M = M^{Tr} + M^{L}$

(2) $\mathrm{U(M)} = \mathrm{U}(M^{Tr} + M^{L}) > 0; \quad U'(M) = \frac{\partial U(M)}{\partial M} > 0; \quad U''(M) = \frac{\partial^2 U(M)}{\partial M^2} < 0$

How does the utility of money (and not the partial indirect utility of obtaining goods) determine the demand for transaction money? All exchange involves giving up one good in exchange for another. The marginal rate of substitution between two goods is the price that measures the rate at which the consumer is just willing to substitute one good for the other. Standard microeconomic theory states that this rate is the slope of the indifference curve which reflects the relative marginal utilities of the two goods at which the consumer is indifferent between holding one or the other good. This logic must apply to money as well.

At the micro-level, the price of a good is the marginal rate of substitution between the commodity and a specified amount of money, and at the margin it reflects the consumer's indifference between holding money or possessing the commodity. The marginal utility of money $U'(M) = \frac{\partial U(M)}{\partial M} > 0$ consists of the *certainty* that it *can* discharge liabilities at any time upon demand. That is the liquidity value of the monetary unit of account. In a monetary exchange, the buyer evaluates the marginal utilities of commodities $\frac{\partial U(y)}{\partial y}$ in view of the ends which they serve, and compares them to the marginal utility of money which defines their exchange value. If the marginal utility of a commodity is higher than the marginal utility of a stipulated amount of money, a purchase will take place. Otherwise, the buyer will keep his money. We then write the price of a good i:

(3) $p_i = \frac{m_i}{y_i} = \frac{\partial U(y_i)}{\partial U(m_i)}$

If the price of the good is £5, it means that the marginal utility of this good is 5 times as high as the marginal utility of keeping £1.

We shift to the macroeconomic level by aggregating the amount of money spent for all transactions:

(4) $\Sigma p_i y_i = \Sigma m_i \;\leftrightarrow\; Py = Y = M^{Tr}$.

which implies that on average the aggregate price level $P = \frac{M^{Tr}}{y}$ reflects all marginal prices.

Transaction money is the aggregate amount of money people plan or expect to spend. The output bought (y) at given prices (p) is the seller's income, so that the total amount of money spent is equal to aggregate income. The stock of transaction money is *proportional* to total expenditure, but to simplify the exposition of my argument I will assume that transaction money is *equal* to aggregate output times the price level, i.e., to aggregate income $(Y = Py = M^{Tr})$. This implies constant velocity of circulation equal to one.[17] We also assume price stability, so that money balances are equivalent to

[17] The Fisher equation, which underlies the quantity theory of money, is $py = M^{T}V$, where V is the velocity of circulation, i.e., the number of turnovers of the given supply of (transaction) money necessary to sell the

real money holdings.[18] Hence, the demand for transaction money determines aggregate income because it is the budget constraint of output:

(5) $M^{Tr} = M(Y) = py$ with $\frac{\partial M^{Tr}}{\partial Y} = 1$, $\frac{\partial M^{Tr}}{\partial y} = p > 0$, $\frac{\partial M^{Tr}}{\partial p} = y > 0$

When transaction money M^{Tr} increases, more output will be sold and income must increase; inversely, if money supply falls, income will fall, too.[19]

1.3. Money, uncertainty, and liquidity preference

In a carefree world, risk neutral people would spend all their money on buying goods and services. Yet, for all kinds of reasons people will not be able to buy all commodities when they wish to acquire them.[20] Although they have reasonable *expectations* about what money can buy, actual spending will vary due to stochastic shocks to prices and output. This is the lottery of a market economy and unfulfilled expectations generate uncertainty and disquietude. To protect themselves against this uncertainty, people will reduce their spending budget and hold precautionary liquid money balances M^L, that serve as an insurance against uncertainty. The aggregate amount of money households plan *not to spend* and hold for security purposes is *precautionary money*. The higher the uncertainty, the more money they will hold back as cash.

We model uncertainty as the error around the planned or *expected average value of spending on transactions.*

(6) $E(M^{Tr}) = E(Y) = \mu_Y + \varepsilon$

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output $V = \frac{Y}{M^{Tr}}$. Setting $V = 1$ yields $M^{Tr} = Y$. The alternative formulation of the Cambridge cash-balance equation is $k = \frac{M^{Tr}}{Y}$ comes to the same result, provided we refer only to transaction money balances. Of course, this is no longer the case if we admit that total money balances consist of transaction and precautionary money holding.

[18] It follows by analogy from equation (3) that the aggregate price level represents the ratio of the average utility of output relative to the average utility of transaction money. In other words, it describes the elasticity by which output reacts to a change in transaction money.

$$P = \frac{U'(y)}{U'(M^{Tr})} = \frac{U(y)/y}{U(M^{Tr})/M^{Tr}}$$

The inverse of the price level measures the purchasing power of money. With price stability this elasticity and purchasing power are constant and equals 1, but in the case of inflation, the reduction of purchasing power lowers the elasticity so that more transaction money generates less output.

[19] To avoid misunderstandings: total spending consists of spending on consumer and investment goods: $M^{Tr} = Y = C + I$, which implies that within a given period saving is always equal to investment because income adjusts. Precautionary money holding is not saving but hoarding.

[20] "Given multilateral credit clearing, a complete set of markets ensuring all contingencies and complete honesty, there would be little need for money. In practise, of course, it is the lack of trust in the counterparties willingness and ability to make promised future payments that makes sellers of spot goods require immediate payment". (Goodhart 1989, 3).

$E(M^{Tr})$ is the *demand of money for transaction purposes*. It is the amount of money required to carry out the budgeted spending plans in an uncertain world, with μ_Y as planned spending budget. The error term ε has zero mean and the variance σ^2. Thus, our measure of uncertainty regarding people's capacity to buy assets is the error variance σ^2.[21] A high variance means a high probability that actual spending could deviate from budgets and consequently people will keep a larger precautionary balance to stay safe. If money supply is fixed, the budgeted spending on goods and services will be reduced and there is a trade-off between *the demand for precautionary* M^L and the demand for planned transaction money M^{Tr}. With given money supply, *transaction money is the residual* left over after risk averse wealth owners decide to hold liquid cash and this determines aggregate output. For this reason, precautionary money holding must not be confused with saving. Saving is a portion of income, but if income falls because money owners keep a larger share of money without spending it, saving will adjust to the new lower income level while the stock of money remains constant. Aggregate income will only grow if aggregate money supply increases more than the demand for precautionary money.

(7) $E(Y) = E(M^{Tr}) = \bar{M} - M^L \quad \rightarrow \quad dE(M^{Tr}) = d\bar{M} - dM^L$

The extent to which people demand holding precautionary liquidity depends on uncertainty, but also on their risk aversion. Risk aversion implies that the utility function of money is concave, i.e., that it has diminishing marginal utility as in equation (2). Risk neutral persons intend to spend all their money on buying goods and services, but risk averse persons prefer to keep some liquidity as safety. This implies that for a risk averse person the utility of the planned budget is higher than the expected utility of average expenditure and budgeted spending is less than the total amount of money supplied. Hence, the never-ending desire to "make money" in the monetary economy does not only depend on greed for goods but also on risk- aversion.

(8) $U[E(M^{Tr}) > E[U(\mu)]$

Different utility functions will yield different degrees of risk aversion. Risk aversion reflects the curvature of the concave utility function. A steeper curvature of the utility function implies higher risk aversion. When risk aversion declines (when the utility curve becomes flatter), and when insecurity diminishes, less precautionary money will be held, and more money will be spent on goods and services.

Risk aversion can be measured by the Arrow-Pratt coefficient θ in absolute and relative form.[22]

(9) Absolut risk aversion: $\theta^{abs} = -\frac{U''(\bar{M})}{U'(\bar{M})} > 0$

Relative risk aversion relates the desire for security, i.e., liquidity preference, to the given stock of money $\bar{M}$.

(10) Relative risk aversion: $\theta^{rel} = -\frac{U''(\bar{M})}{U'(\bar{M})}\bar{M} = \theta^{abs}\bar{M} > 0$

Because both coefficients are functions of the aggregate money stock, we can write (9) also as

[21] Strictly, the expected uncertainty is the conditional variance, but to keep things simple we use the notation σ^2 for the conditional variance.

[22] (Arrow 1970); (Pratt 1964)

(11) $\theta^{abs} = \rho(\bar{M})$ with $\rho'(\bar{M}) = \frac{\partial \theta^{abs}}{\partial \bar{M}} = -\frac{U'(\bar{M})U'''(\bar{M}) - [U''(\bar{M})]^2}{[U'(\bar{M})]^2}$

We can now determine precautionary money demand, which is roughly proportional to absolute risk-aversion and the measure of uncertainty:[23]

(12) $M^L = f[\rho(\bar{M}), \sigma^2] \approx \frac{1}{2}\rho(\bar{M})\sigma^2$

With

(13) $\frac{\partial M^L}{\partial \bar{M}} = \frac{1}{2}\rho'(\bar{M})\sigma^2$ and $\frac{\partial M^L}{\partial \sigma^2} = \frac{1}{2}\rho(\bar{M})$

Equation (12) describes liquidity preference as a function of risk aversion and uncertainty. The total change in the demand for precautionary money balances is

(14) $dM^L = \frac{\sigma^2}{2}\rho'(\bar{M})d\bar{M} + \frac{1}{2}\rho(\bar{M})d\sigma^2$

which implies that the demand for *precautionary money holding will unequivocally increase with uncertainty* (because $\rho(\bar{M}) > 0$), but the *effect of a change in money supply* depends on the sign of $\rho'(\bar{M})$. It seems reasonable to assume that absolute and relative risk aversion will diminish as money supply increases, because people will spend more money on transactions when their desire for security is satisfied. This means $\rho'(\bar{M}) < 0$, which is the case when the third derivative of the utility function is positive, i.e., when the rate at which the marginal utility diminishes remains positive.[24] Some authors have called this condition the prudence criterion (Eeckhoudt und Schlesinger 2006).

Many textbooks relate risk aversion to non-monetary variables such as consumption or wealth and assume isoelastic utility functions, which yield elegant mathematical results.[25] An isoelastic utility function for money would yield constant relative risk aversion (CRRA) meaning that people will hold the same percentage of total money in the form of liquidity as money supply increases. Given stable prices, the long run the growth of money supply would proceed at the same rate as output. This is the monetarist interpretation of money (Friedman 1956). However, this does not seem reasonable. Only *for a given level of uncertainty* will people increase spending when money supply grows because the stock of precautionary money covers their demand for security. Furthermore, constant relative risk aversion *with respect to money* would prohibit any form of monetary policy because it would imply

[23] See (Rao and Jelvis 2021?, chapter 5) for the math which takes too much space to be reproduced here.
[24] See equation (11) and remember that $U''(M) < 0$.
[25] The isoelastic utility function, which yields constant relative risk aversion (CRRA), is the "canonical form found in all graduate textbooks" (Aghion and Howitt 2009, 37). It has the form

$$U(x) = f(x) = \begin{cases} \frac{x^{1-\theta} - 1}{1-\theta}, & \theta \geq 0, \quad \theta \neq 1 \\ \ln(x), & \theta = 1 \end{cases}$$

that increasing money supply would not affect people's liquidity preference.[26] I will therefore work with the assumption that $\rho'(\bar{M}) < 0$.[27]

From equation (6) and (7) we know that with given money supply the demand for transaction money (spending on goods) will be reduced as precautionary money increases. Thus, an increase in uncertainty will lower spending and output, but an increase in money supply will raise spending and income (because $\rho'(\bar{M}) < 0$).

$$(15)\quad dE(M^{Tr}) = \left(1 - \frac{\sigma^2}{2}\rho'(\bar{M})\right)d\bar{M} - \frac{1}{2}\rho(\bar{M})d\sigma^2$$

The trade-off between precautionary and transaction money (equation 7) allows us to define the liquidity premium, which a person would charge a compensation for giving up liquidity. In commercial spot contracts (cash purchases) the compensation for giving up liquidity is the utility of commodities. In financial contracts the borrower is promising to compensate the lender in the future, while he obtains the advantage of having immediate access to money. The financial contract therefore generates a claim (financial asset) as well as an obligation (liability) for future payment and it determines the compensation for *giving up the possession of money*. The amount of the contractual consideration is defined in terms of currency units; it is obviously the same for borrower and lender in each period because of the flow of funds identity. However, the terms of the contract reflect the price the borrower must pay for obtaining liquidity now. The willingness by the lender to give up the possession of money is measured by the liquidity premium which is determined as:

$$(16)\quad l = \frac{M^L}{\bar{M}} \approx \frac{1}{2}\frac{\rho(\bar{M})}{\bar{M}}\sigma^2 \text{ with } \quad \frac{\partial l}{\partial M} = \frac{\frac{\sigma^2}{2}\rho'(\bar{M}) - \rho(\bar{M})}{\bar{M}^2} < 0, \qquad \frac{\partial l}{\partial \sigma^2} = \frac{1}{2}\frac{\rho(\bar{M})}{\bar{M}} > 0$$

For given money supply, the liquidity premium raises in proportion to uncertainty and the impact depends on relative risk aversion. An increase in money supply lowers the liquidity premium (because $\rho'(\bar{M}) < 0$), but the effect becomes weaker as the quantity of money increases (this can cause Keynes's liquidity trap). In the short run changes in money supply can stabilise income by compensating shocks caused by uncertainty.[28]

The liquidity premium reflects the proportion of a given money stock that risk averse wealth owners wish to hold in liquid form in an uncertain environment. That sets the conditions in the credit/bond

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[26] (Benchimol 2014) has worked with a CRRA in assessing the impact of monetary policy on optimal consumption in the Euro Area but he allowed stochastic shocks for its variation. He found that in a standard New Keynesian DSGE model a risk aversion shock increases inflation, decreases output and diminishes the impact of the action by the central bank on output variance. Risk aversion plays a negative role in determining output, whereas it increases real money balances and real money growth in the initial period. (Friend and Blume 1975) provided evidence that investors require a substantially larger premium to hold equities or other risky assets than they would if their attitudes toward risk were described by logarithmic utility functions.

[27] It could be argued, however, that when money was coinage made of bullion, money supply depended on the trade balance which is closely correlated with transaction money, so that $\rho'(\bar{M}) = 0$. But of course, in ancient times there was no monetary policy.

[28] In the very long run, the liquidity premium may be stationary. Augmented Dickey-Fuller Test indicate stationarity from 1763 to 2016 for English interest rates from 1763 to 2016 for Bank rates, consols and prime commercial paper. Author's estimates based on (Bank of England 2017).

market.[29] The interest rate is the equilibrium price at which money can be borrowed. Making loans (buying bonds) means giving up liquidity for a limited period, and the liquidity premium is the exogeneous component. However, other components also contribute to this price. In an economy with financial intermediaries, the interest rate also compensates commercial banks for the cost of obtaining currency units from the central bank (d) and for the risks of specific asset losses (γ) and other carrying costs (δ).

(17) $r = l + d + \gamma + \delta$

Historically these components have played different roles. When coinage was money and there was no central bank, the discount rate d was the difference between the face value of coins and the value of the metal they were made of. With metal currency, carrying costs δ were substantial. Mediaeval usury laws prohibited charging the liquidity premium l, but they did not consider that charging for the potential risk of loss γ and carrying costs δ was immoral and sinful.[30] Usury meant charging for liquidity, because it implied that the lender would receive more than had been given away, and this exchange of something for nothing contradicted the scholastic concept of "just price". The abolition of usury meant that the liquidity premium became the critical component for the determination of interest. The creation of credit money after the foundation of the Bank of England in 1694 allowed more flexible money supply, which helped to reduce interest rates. However, because monetary policy reacts to exogenous shock, I will continue my argument as if the liquidity premium were the only component in the interest rate. This implies that uncertainty will raise, and money supply will lower the interest rate.[31]

2. The dynamics of economic growth

Having derived the interest rate in the context of liquidity preference that determines the *form in which households wish to hold their wealth*, we turn to the issue of *how much income* does a person wish to consume now versus saving for consumption in the future.[32] We assume a simple intertemporal choice model with two time periods. There are N individuals owning the initial money stock $\bar{M}_t$. They all generate an equal amount of income in the first but not necessarily in the second period. At the end of the second period all income is consumed. Within each period, saving is equal investment, and this identity is the result of adjustments in aggregate income. Investment generates higher income with diminishing returns. Solvency of firms requires that the return on capital must match the exogenously set interest rate. The gap between the marginal productivity of capital and the interest rate is filled by technological progress.

2.1. Saving and intertemporal exchange

We write Y_t for income and C_t for consumption and index $t = 1$ for the first period (today), and $t = 2$ for the second (tomorrow). In each period spending is income and planned spending is equal to the amount of transaction money $\mathrm{E(Y)}_t = \mathrm{M}_t^{Tr}$. Thus, in accordance with (7), aggregate spending in each

[29] Interest rates do not *determine* the amount of money wealth owners wish to hold liquid as opposed to in other assets, such as bonds; they are the price of money they wish to hold as a protection against uncertainty which determines interest rates.
[30] See (Kerridge 1988)
[31] This is equivalent to saying that I abstract from the banking system. In that case, the liquidity premium is the equilibrium interest rate at which the borrower's demand for money equals the lender's supply.
[32] Keynes (1967 [1936], 166)

period is constrained by transaction money, i.e., by the difference between money supply and liquidity preference and over the two-period lifetime, total spending on consumption is equal to output:

$$(18)\ (\mathrm{M}_1^{Tr} + \mathrm{M}_2^{Tr}) = (C_1 + \mathrm{C}_2) = (\mathrm{Y}_1 + \mathrm{Y}_2)$$

However, within each period, income and consumption are not equal. Saving is defined as the positive difference between income and consumption, a negative balance is borrowing. Yet, because all income is consumed at the end of period 2, there are no savings left over and the capital stock is constant.[33]

$$(19)\ S_t = \mathrm{M}_t^{Tr} - \mathrm{C}_t = \mathrm{Y}_t - \mathrm{C}_t$$
$$(20)\ (\mathrm{Y}_1 + \mathrm{Y}_2) - (C_1 + \mathrm{C}_2) = S_1 + S_2 = 0$$

We now distinguish two groups, lenders and borrowers. In the flow of funds literature households are lenders and firms are borrowers. We can simply assume that lending and borrowing is mediated by banks for individuals who have identical utility functions and initially generate equal shares of aggregate income. Saving S by one group is the borrowing B by the other group. Borrowing means issuing a bond and borrowers spend the borrowed money today.[34] In a pre-capitalist economy, lenders' savings finance contemporary consumption of borrowers. In a capitalist economy they finance investment, which is the contemporary net addition to the capital stock: $S_1^L = B = I_1^B$ and is intended to generate additional income tomorrow. However, either way *all transaction money is spent on aggregate output of the same period*. Money that is not spent is precautionary money and not saving. The lender reduces her consumption and transfers the saved part of her transaction money to the borrower who then uses this money to buy the saved part of her income. Thus, aggregate output (income) is absorbed ("consumed") by the transaction money of each period.

$$(21)\ \mathrm{M}_1^{Tr,Le} = \mathrm{Y}_1^{Le} - S_1^{Le} = C_1^{Le} \qquad \text{lender}$$
$$(22)\ \mathrm{M}_1^{Tr,B} = \mathrm{Y}_1^B + S_1^{Le} = \mathrm{Y}_1^B + B = \mathrm{Y}_1^B + I_1^B \qquad \text{borrower}$$

Hence, in the first period saving is equal investment:

$$(23)\ S_1^{Le} - I_1^B = 0 \ \text{ and}$$
$$(24)\ \mathrm{M}_1^{Tr} = \mathrm{M}_1^{Tr,Le} + M_1^{Tr,B} = \mathrm{E}(\mathrm{Y}_1)$$

We also define the saving ratios s_t':

$$(25)\ s_1' = \frac{S_1^{Le}}{Y_1^{Le} + Y_1^B} = \frac{S_1}{Y_1} \qquad \text{and} \qquad s_2' = \frac{S_2}{Y_2}$$

History provides three models for relating the two periods. First, for an *autonomous household* with a given amount of income, there is no exchange and therefore no money.[35] Intertemporal transfer of wealth means, the household can decide how much of its real income it will save now to consume it

[33] For ease of presentation, I drop the expectation term. We could also assume real income as a continuum of goods as in (Dornbusch, Fischer and Samuelson 1977), so that the non-consumed income of this period is not necessarily left over for consumption in the next period.
[34] I model the relation between households and non-financial corporations as described in the flow of funds literature. This is not an overlapping generations model because lenders and borrowers spent their income in the same period.
[35] This model also applies to non-monetary palatial societies, where reserve stocks were kept in the palaces. Examples are ancient Mesopotamia, Egypt, Crete, the Incas in Peru, or Angkor in Cambodia.

tomorrow. If saving means not having dinner today,[36] this transfer implies that the household will eat the leftovers tomorrow. This is a way of transferring output from period 1 to period 2, but aggregate income over the two periods remains unchanged. If there are carrying costs that depreciate the saved output, the rate of depreciation would be the discount rate.

Second, in *precapitalist exchange economies*, money existed in the form of coinage and people lent and borrowed from each other, but they did not charge interest. We may call this *charity lending*. In period 1, the lender consumes less than the borrower. The borrower's additional consumption is the lender's saving. In period 2 the relation is inversed. The borrower repays the lender by saving from his income in period 2. Without interest the logic is "I share my food with you today and you share yours tomorrow".[37] Charity lending works because the motivation for lending does not follow economic but moral criteria.

Third, in a *capitalist economy*, credit contracts carry interest obligations. Assuming the money stock is given, the saver lends her savings S_1^{Le} to the borrower if she gets compensated for her liquidity preference.[38] The borrower issues a bond that promises to pay back principal B plus interest at the rate r in period 2. The lender "buys" the bond and therefore transfers her money to the borrower. In exchange, she receives the promise to get this money back *plus interest*, and the borrower has a liability in period 2 which exceeds the value of the principal by the amount of interest: $B(1+r)$.

The credit contract distorts the initial equality of income of our model, because the lender will receive a larger amount of money and income in the future. She has given away her savings $S_1^{Le}=B$, but in the next period she gets it back plus interest $B(1+r)$. The borrower is enabled to spend more than his income today, but the saver has a claim on the borrower that will enable her to spend *more* tomorrow. Contrary to the case of charitable lending, in the capitalist economy the lender's spending capacity in period 2 is not only supplemented by her previous savings, but she receives additional money income in the form of interest from the debtor. The borrower, on the other hand, will use the loan in period 1 either to increases his consumption or to invest it to produce more. If we assume that aggregate income will be constant over the total period, the borrower must *reduce consumption* in period 2 by more than what he had borrowed in the previous period and the lender will consume more.[39] Alternatively, if he borrows to invest, i.e., increases the capital stock, total income will no longer be constant, and the *economy will grow*. Hence:

(26) $S_1^{Le}=B;\ S_2^{B}=(1+r)B$

Equation (26) must be understood in terms of the flow of funds identity in the balance sheets of lenders and borrowers. By agreeing on a credit contract in period 1, the lender has obtained a contractual claim on the borrower that is her financial asset, while the borrower has committed

[36] (Keynes 1967 [1936], 210)

[37] The prevalent form for providing subsistence security in Ancient Greece was through interest-free *eranos*-loans, which were seen as a contribution to the community. In the pre-monetary economy described by Homer, ἔρανος meant *meal to which each contributed his share, picnic.* See (Millett 1991, 39; 154-55).

[38] I have argued in the previous section that the liquidity premium is the equilibrium interest rate in a model without financial intermediation. However, in a system with banks, the equilibrium interest rate also reflects the cost at which banks obtain cash from the central bank.

[39] This explains the rationality of usury laws in ancient economies. Income was largely dependent on agriculture with low elasticity of supply and money was constrained by the metal base of coinage. Given that a large part of the population lived close to subsistence income levels, the risk of borrowers falling into destitution was high.

himself to an equivalent liability. *In period 2, the liability is increased by the contracted interest obligation.* Thus, to avoid bankruptcy, the borrower must generate higher assets to match the increased liability and produce the income he needs to repay his debt. In principle, he can do so by either reducing consumption and increasing saving in period 2 or by increasing output. However, if the reduction of consumption is constrained by a minimum income level required for subsistence, the borrower's only option to prevent destitution is to increase output. This follows from the *solvency constraint* inherent in balance sheets. He can reduce consumption or increase income. For poor people living close to the subsistence limit, the reduction in consumption is not a sustainable strategy.[40] The solvency constraint is:

$$(27)\ S_1^{Le} = \frac{S_2^B}{1+r} = \frac{Y_2^B - C_2^B}{1+r}$$

By referring to the definition of the saving ratios (25) and the solvency constraint (27) we get

$$(28)\ s_2' Y_2 = (1+r) s_1' Y_1$$

Which yields the intertemporal rate of transformation of present into future consumption

$$(29)\ \frac{C_2}{C_1} = \frac{s_2' Y_2}{s_1' Y_1} = (1+r)$$

Optimal consumption smoothing over the two periods implies that the marginal rate of substitution between C_1 and C_2, which is the ratio of the two respective marginal utilities, must be equal to the rate of transformation.

$$(30)\ \frac{U'(C_2)}{U'(C_1)} = (1+r) \qquad \leftrightarrow \qquad U'(C_1) = \frac{U'(C_2)}{(1+r)}$$

Equation (30) is the Euler equation that characterises the first-order condition of intertemporal optimal choice which equates the marginal cost of giving up consumption today for the marginal benefits of receiving higher income and consumption tomorrow. It implies that the utility of present consumption must be equal to the discounted utility of future consumption. The discount rate contains the exogenous component of the liquidity premium. Hence the rate at which present consumption is traded off for future consumption depends on uncertainty and risk aversion.

In neoclassical growth models, profit maximising firms invest until the diminishing marginal productivity of capital has reduced profits to the level of the cost of capital. Capital accumulation raises income until the stock of capital is constant in the steady state and the asset side of the firm's balance sheet is the driver of economic growth. However, in the analysis suggested in this paper, it is the liability side that drives growth, because the cost of capital is exogenously determined by uncertainty, risk aversion and the liquidity premium. In the steady state, liabilities grow exogenously at the same rate as the interest rate and assets must match this growth. We know from equation (29) that the optimal consumption path which meets the solvency constraint requires that income grows at this rate, too. Thus, the rate of balanced growth is the rate of interest. However, if the capital stock - or

[40] After the liberalisation of interest taking in the 16th century by Elizabeth I, Puritanism developed rapidly in England which focused on both increasing income and reducing consumption, which, as Max Weber (1958 [1922]) has argued, initiated modern capitalism.

more precisely the capital-labour ratio – is constant in the steady state, then output can only be increased by higher factor productivity, i.e., technological progress.

2.2. The warranted growth rate and technological progress

We now give up the assumption that income is constant over time. How much does income have to grow to satisfy the solvency constraint? We define income growth as

$$(31)\quad \frac{Y_2}{Y_1} = 1 + g$$

Using equation (28) and (31) we have

$$(32)\quad s_2' = \frac{S_2}{Y_2} = \frac{(1+r)S_1}{Y_2} = \frac{(1+r)}{(1+g)} s_1' \leftrightarrow \frac{s_2'}{s_1'} \equiv \left(1 + \frac{\Delta s'}{s_1'}\right) = \frac{(1+r)}{(1+g)}$$

We define the growth rate warranted by the solvency constraint as the *warranted growth rate*:

$$(33)\quad g_w = \left(\frac{1}{1+\frac{\Delta s'}{s_1'}}\right)\left(r - \frac{\Delta s'}{s_1'}\right)$$

Thus, for constant saving rates $\frac{\Delta s'}{s_1'} = 0$ we get

$$(34)\quad g_w = r$$

However, an increase in saving will lower the warranted growth rate required for solvency. Equation (34) tell us that, given the solvency constraint, the savings ratios can only remain constant if the economic growth rate is equal to the interest rate. Most economic growth models assume a constant saving rate and empirical evidence points to long run stationarity in this rate. Changes in the saving rate are then reflecting stochastic shocks, but its mean is constant. Hence, in the long run the warranted growth rate must equal the interest rate. However, *if the interest rate is exogenously determined, then the mean economic growth rate must endogenously adjust to this rate*. Otherwise, the solvency constraint is violated and borrowers will default.

This raises the question what will ensure that income growth satisfies the solvency constraint. I will discuss that below. First, however, a remark on the controversial relation between the interest and the growth rate. According to capital market equilibrium theory, long-term real interest rates are determined by long-term trends in saving and investment. Real interest rates fall when there is an excess of saving over investment. Alternatively, a low real interest rate may reflect a lack of productive investment opportunities in the economy. However, as Keynes has shown, saving and investment depend on aggregate income, and it is income and not the interest rate that ensures the equality of saving and investment. In the long run, dynamic efficiency should ensure that the interest rate is equal to the growth rate.[41] Monetary policy is supposed to have only short-term effects but does not determine long run interest rates. This view is not compatible with the Keynesian view that the interest rate is exogenously determined by liquidity preference. However, assuming a stable utility function, we have two exogenous factors determining the interest rate in the Keynesian model: one is uncertainty, the other is money supply which is controlled by monetary authorities.[42] Uncertainty is

[41] At least this is true for abstract models (Blanchard and Fisher 1989). Empirical verification is more difficult (Abel, et al. 1989).

[42] Because we model a closed economy, we can ignore balance of payment movements.

an exogenous shock that pushes up the liquidity premium, but increasing money supply will lower the interest rate because it reduces risk aversion, as we have seen above.

Now, what is required for actual growth to match the warranted rate? We can explain the growth rate of aggregate income by using a standard neoclassical growth model with technological change.[43] I do not wish to explain technological change as such, but simply show that if the interest rate is exogenously determined by uncertainty, technological progress will ensure that the solvency condition is satisfied even in the steady state.[44]

We start with a standard Cobb-Douglas production function. All variables are indexed on time t, although I do not show this explicitly to keep readability simple.

(35) $Y = (AL)^{1-\alpha}K^{\alpha}$

Where K is the capital stock, A is a productivity indicator and AL is the "effective supply of labour", which grows at the rate of population growth $\hat{n}$ and the growth rate of productivity $\hat{a}$. Technological *progress* $\frac{dA}{dt}/A = \hat{a}$ reflects all factors, such as technological improvements, better skills, more efficient institutions, etc., that allow increasing output without having to save and invest more or using more resources. Capital per efficiency unit is

(36) $k = \frac{K}{AL}$ with the growth rate $\hat{k} = \frac{dk/dt}{k}$

And output per efficiency unit is

(37) $\varphi = \frac{Y}{AL} = k^{\alpha}$ with $\alpha < 1$ because of diminishing marginal productivity of capital

Saving raises the k-ratio, while depreciation lowers it. Investment is the net increase in capital with δ the rate of depreciation.

(38) $I = \frac{dK}{dt} = sY - \delta K.$

Therefore, the rate of change in k is

(39) $\frac{dk}{dt} = sk^{\alpha} - (\hat{n} + \hat{a} + \delta)k$

In the short run, investment may increase with a reduction in interest rates because new profitable opportunities arise. In the long run, however, k will approach a unique steady state value k^* and φ approaches the steady state $\varphi^* = (k^*)^{\alpha}$. Although output per efficiency unit does not grow in the long run, the same is not true for output per person.

(40) $\frac{Y}{L} = A\varphi = Ak^{\alpha}$

[43] See (Aghion and Howitt 2009, 27-29)

[44] Solow (2000, 119; 149) has pointed out that exogenous growth depends only on technological and demographic parameters, while "endogenous growth means output should be growing faster than the exogenous factors alone can make it grow". In my model, the exogeneous factors are the liquidity premium (uncertainty) and population growth, while capital accumulation is a parameter in the steady state.

The growth rate of output per person is then

(41) $g - \hat{n} = \hat{a} + \alpha\hat{k}$

Given that in the long run k approaches the steady state $k^*$where it no longer changes, the growth rate $\hat{k}$ tends to zero, so that in the steady state per capita income will grow at the exogeneous rate of technological change and total income grows at the rate of technical progress plus population growth.

Combining (34) with (41) we get the equilibrium condition

(42) $g_w = r = \hat{n} + \hat{a}$

In order to satisfy the solvency constraint, the economy must grow at the exogenously determined rate of interest which requires that a larger population and technological progress generate the income necessary to service the outstanding debt. Firms that borrow money to invest are under pressure to increase their productivity to avoid going bankrupt. This is the major driver of growth. Technological progress is the consequence of the monetary credit economy.

2.3. Monetary policy implications

The critical component in our model is the interest rate which contains the exogenous component of the liquidity premium and a policy variable, which is the central bank rate for money supply. Under conditions of balanced growth income and money grow at the rate of interest and prices are stable. Price stability requires that transaction money grows at the rate of planned expenditure (see equation 7), but planned expenditure depends on how much money people hold in liquid form and on how much money the banking system provides. In a stable environment of uncertainty and risk aversion, an increase in total money supply will lower the liquidity premium (equation (16)), and precautionary money will be shifted into transaction money, so that higher output will be bought. Hence, increasing money supply and lowering interest rates will raise the growth of economic income and saving and therefore the warranted growth rate required for solvency. However, a positive gap between the actual and the warranted rate would generate inflation, which would force the central bank to tighten monetary policy and rise interest rates. In a stable environment, one should therefore expect the liquidity premium and interest rates to be stationary. When this stability is undermined, say because of uncertain wars or political uncertainties, our measure for uncertainty - the conditional variance σ^2 – will be raise and less transaction money is available. The central bank would then aim to stabilise output by increasing money supply. Ideally this increase in money supply is absorbed by additional precautionary money holdings, so that prices remain stable. But there is no automaticity that this will be the outcome.

3. Empirical evidence of interest rates on economic growth

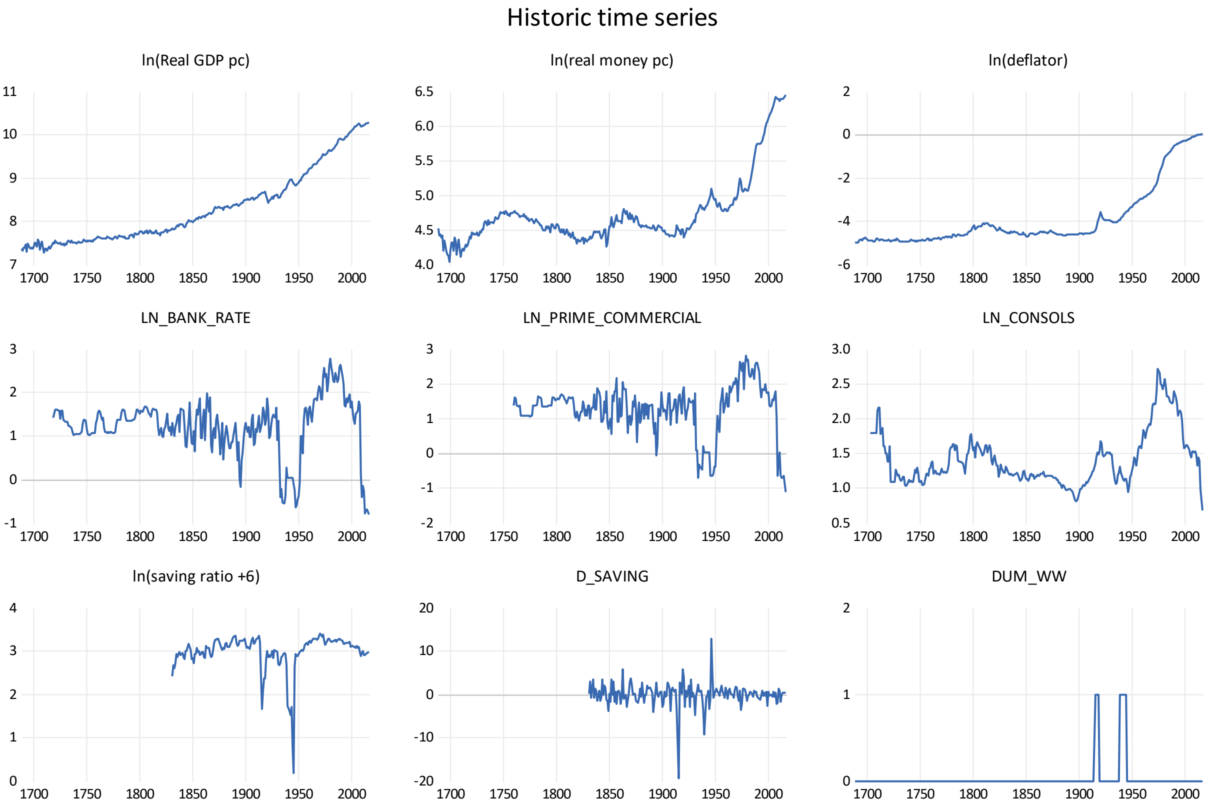
We can now look at some empirical evidence. We are interested whether the data confirm a positive relation between exogenous interest rates and economic growth, taking into consideration variations in money supply and saving. As with many other economic variables, some basic economic conjectures will empirically hold only over very long time horizons.[45] In this last section, I shall therefore provide some evidence from Great Britain covering 182 years.

[45] See for example (Taylor 2002).

3.1. The data

We first look at evidence from the historic data series provided by the Bank of England.[46] To test our model against we take real time series for GDP per capita, broad money per capita, and the GDP deflator. Unfortunately, the time series for savings ratio only starts in 1830. It is stationary with a mean of 15%, although during the two World Wars saving rates became negative. I therefore add a dummy for the two World Wars and to get positive values for taking logarithms, I add a constant number 6 to each value before taking logarithms. We also find three series for interest rates in the Bank of England database: the (spliced) bank rate, prime commercial paper, and yields on perpetual annuities (Consols). All nominal variables are converted into real rates by using the GDP deflator and we take logs for all timeseries. Figure 1 shows the data.

Figure 1.



3.2 Estimation

The logic of our model hinges on the exogenous nature of interest rates that are a function of liquidity preference and uncertainty. We must therefore identify which rates are exogenous. Granger causality tests (see Annex 1) show that real GDP granger-causes money supply, GDP deflator, bank rate and prime commercial paper rates, but not yields on consols. Inflation causes changes in money supply (but not the inverse), while money supply impacts the prime commercial and banking rates. The savings ratio granger-causes inflation, bank rates, and prime commercial paper rates, while the prime rate also influences bank rates. The yields on perpetual annuities (consols) play a particular role. They granger-cause money supply and inflation, but inflation also granger-causes yields on consols. There

[46] (A millennium of macroeconomic data for the UK. Version 3.1 data finalised 30 April 2017 based on Blue Book 2016 with some historical extensions to version 3.0 made in S 2017)

is a 28.5 percent probability that consols do not granger-cause GDP, and a 69.2 percent probability that GDP does not cause yields on consols. Hence, there is an interaction between consols yields and GDP, but the impact from interest rates to GDP is nearly 2 ½ times more likely than in the opposite direction. Thus, we are justified to assume that consols are the measure for our exogenous interest rates that reflects liquidity preference in markets, while bank rates and primary commercial paper are reflecting more directly monetary policy measures (via the central bank-controlled discount rate).

To evaluate the impact of consoles yields on GDP, I estimate a non-linear autoregressive distributed lag (NARDL) model (Shin, Yu and Greenwood-Nimmo 2014), following the methodology described in the Eviews blog (IHSEViews 2022). I allow for separate impacts for the variations of money supply, deflator, consol yields and saving ratio.[47] Unit root tests confirm that our variables are I(1) in levels (See Annex 2).

The bound test for cointegration confirms a long run cointegration relation (see Annex 3), and the symmetry test (Annex 4) rejects the null-hypothesis of symmetric effects for consols in the short and long run, but for inflation only in the long run. Thus, cutting interest rates does not have the same effect as increasing them. Money supply and saving have symmetric effects. The fully estimated model is shown in Annex 5.

We are interested in the long run cointegrating coefficients shown in Table 1. The effect of changes in money supply is statistically not significant, but clearly positive for saving. The coefficients for consoles, by contrast, are asymmetric and statistically significant at 1%. As standard textbook economics would expect, a reduction in interest rates increases GDP. We find that indeed, a reduction in consol rates (CUMD**N)** increases economic growth in the long run and this effect is 37.5 percent.[48] However, most interestingly, and in line with our theoretical model, an *increase in the interest rate increases* growth in the long run, too. Here, the effect is 29.1 percent. An increase in inflation has a negative effect on growth (within the 10 percent significance range), but the effect from lower inflation is uncertain (although with the right sign).

Table 1. Cointegrating coefficients

Variable *	Coefficient	Std. Error	t-Statistic	Prob.
LN_MONEY(-1)	0.096756	0.086733	1.115563	0.2661
LN_SAVING(-1)	0.343228	0.047387	7.243148	0.0000
@CUMD**P**(LN_CONSOLS(-1))	**0.291287**	0.087707	3.321119	**0.0011**
@CUMD**N**(LN_CONSOLS(-1))	**-0.375222**	0.070841	-5.296711	**0.0000**
@CUMDP(LN_DEFLATOR(-1))	-0.129527	0.074671	-1.734633	0.0845
@CUMDN(LN_DEFLATOR(-1))	-0.026964	0.072127	-0.373842	0.7090
C	6.455643	0.431553	14.95910	0.0000

Note: * Coefficients derived from the CEC regression.

The effect of a shock in one of the model's variables can be shown by the cumulative dynamic multipliers. The effect of a shock to money supply is initially high and then gradually converges down to the long run equilibrium. Inversely, an increase in saving builds up gradually over time. See Figures

[47] CUMD**P** stands for the positive cumulative change in the variable, CUMD**N** for the negative value.

[48] Because the coefficient for negative changes in consoles (CUMD**N)** is negative, the effect is positive.

2 and 3. The effect of inflation is uncertain (the confidence interval remains in the zero range). See Figure 4.

Figure 2. Money effect

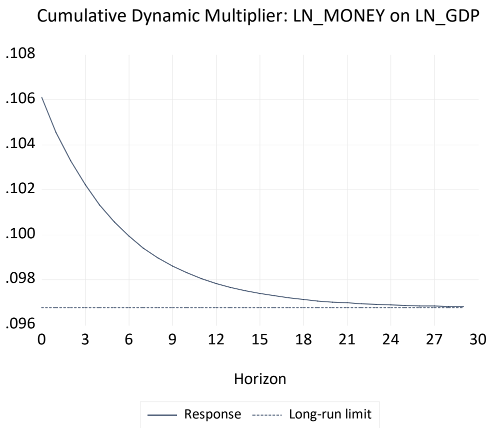


Figure 3. Saving effect

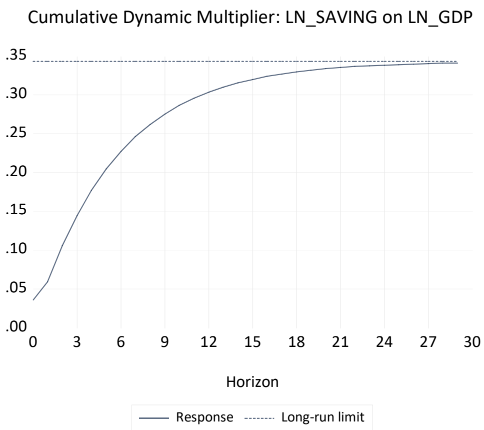
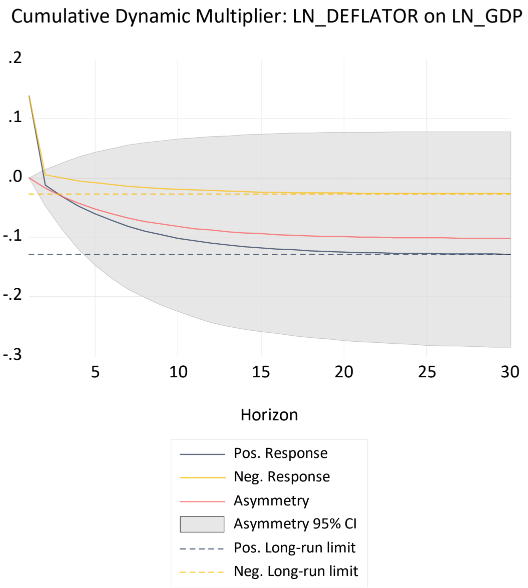
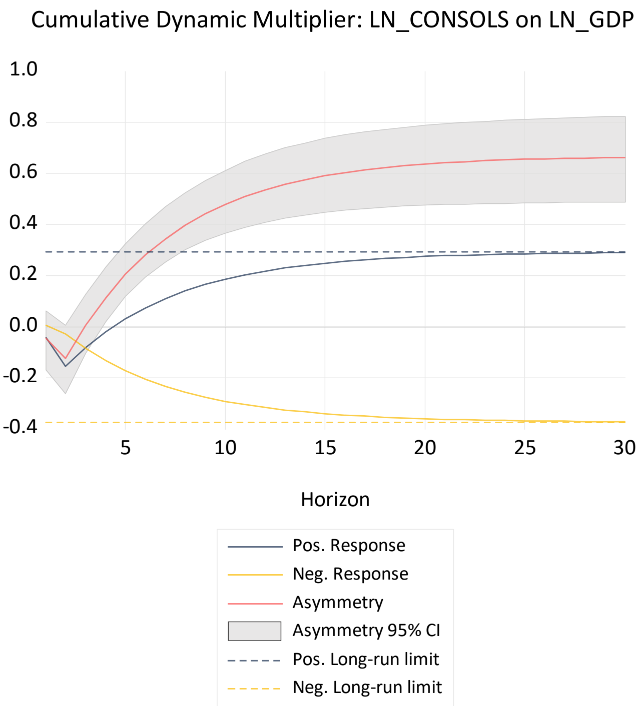


Figure 4.



However, most importantly, the effect of consol rates, which is our proxy for the exogenous interest rate, is not significant in the short run below 4 years. After that period, it becomes clearly positive and highly significant.

Figure 5.



The effect of interest rates is asymmetric for rate rises and cuts. Standard economic theory explains that a cut in interest rates increases growth because at lower borrowing costs investment projects with lower marginal productivity become attractive. In Solow-growth models this implies that the economy is not in the steady state. I have argued, however, that in the steady state an increase of interest rates forces economic growth because firms must increase output to respect the solvency constraint. Assuming that the yields on consoles contain a significant portion of the liquidity premium, a rise in in this rate leads to an increase in GDP. The econometric evidence validates both explanations. Our model, therefore, presents credible evidence that interest rates reflecting the exogenous conditions of uncertainty, risk averseness and liquidity preference, are indeed the driver of economic growth. QED.

Annex 1

Pairwise Granger Causality Tests
Date: 10/11/22 Time: 19:35
Sample: 1688 2016
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
LN_REAL_MONEY_PC does not Granger Cause LN_REAL_GDPPC	327	1.77651	0.1709
LN_REAL_GDPPC does not Granger Cause LN_REAL_MONEY_PC		6.16553	**0.0024**
LN_DEFLATOR does not Granger Cause LN_REAL_GDPPC	327	2.25092	0.1070
LN_REAL_GDPPC does not Granger Cause LN_DEFLATOR		6.34404	**0.0020**
LN_BANKRATE does not Granger Cause LN_REAL_GDPPC	297	1.35512	0.2595
LN_REAL_GDPPC does not Granger Cause LN_BANKRATE		9.86439	**7.E-05**
LN_CONSOLS does not Granger Cause LN_REAL_GDPPC	312	1.26186	*0.2846*
LN_REAL_GDPPC does not Granger Cause LN_CONSOLS		0.36925	*0.6916*
LN_PRIME_RATE does not Granger Cause LN_REAL_GDPPC	255	1.03575	0.3565
LN_REAL_GDPPC does not Granger Cause LN_PRIME_RATE		10.5415	**4.E-05**
SAVING does not Granger Cause LN_REAL_GDPPC	185	2.03660	0.1335
LN_REAL_GDPPC does not Granger Cause SAVING		0.33038	0.7191
D_SAVING does not Granger Cause LN_REAL_GDPPC	184	1.09940	0.3353
LN_REAL_GDPPC does not Granger Cause D_SAVING		1.06498	0.3469
LN_DEFLATOR does not Granger Cause LN_REAL_MONEY_PC	327	12.3369	**7.E-06**
LN_REAL_MONEY_PC does not Granger Cause LN_DEFLATOR		1.24381	0.2897
LN_BANKRATE does not Granger Cause LN_REAL_MONEY_PC	297	2.52415	*0.0819*
LN_REAL_MONEY_PC does not Granger Cause LN_BANKRATE		4.00295	**0.0193**
LN_CONSOLS does not Granger Cause LN_REAL_MONEY_PC	312	6.23934	**0.0022**
LN_REAL_MONEY_PC does not Granger Cause LN_CONSOLS		1.01146	0.3649
LN_PRIME_RATE does not Granger Cause LN_REAL_MONEY_PC	255	1.13304	0.3237
LN_REAL_MONEY_PC does not Granger Cause LN_PRIME_RATE		6.16638	**0.0024**
SAVING does not Granger Cause LN_REAL_MONEY_PC	185	0.97176	0.3804
LN_REAL_MONEY_PC does not Granger Cause SAVING		1.16018	0.3158
D_SAVING does not Granger Cause LN_REAL_MONEY_PC	184	0.96995	0.3811
LN_REAL_MONEY_PC does not Granger Cause D_SAVING		1.51979	0.2216
LN_BANKRATE does not Granger Cause LN_DEFLATOR	297	0.12410	0.8833
LN_DEFLATOR does not Granger Cause LN_BANKRATE		2.96933	**0.0529**
LN_CONSOLS does not Granger Cause LN_DEFLATOR	312	3.26842	**0.0394**
LN_DEFLATOR does not Granger Cause LN_CONSOLS		6.34165	**0.0020**
LN_PRIME_RATE does not Granger Cause LN_DEFLATOR	255	0.03912	0.9616
LN_DEFLATOR does not Granger Cause LN_PRIME_RATE		1.60710	0.2025
SAVING does not Granger Cause LN_DEFLATOR	185	0.32278	0.7246
LN_DEFLATOR does not Granger Cause SAVING		0.52592	0.5919
D_SAVING does not Granger Cause LN_DEFLATOR	184	9.63392	**0.0001**
LN_DEFLATOR does not Granger Cause D_SAVING		0.74488	0.4763
LN_CONSOLS does not Granger Cause LN_BANKRATE	297	2.32887	0.0992
LN_BANKRATE does not Granger Cause LN_CONSOLS		1.55494	0.2129

LN_PRIME_RATE does not Granger Cause LN_BANKRATE	255	51.9153	**1.E-19**
LN_BANKRATE does not Granger Cause LN_PRIME_RATE		4.34690	**0.0139**
SAVING does not Granger Cause LN_BANKRATE	185	4.23855	**0.0159**
LN_BANKRATE does not Granger Cause SAVING		0.50201	0.6062
D_SAVING does not Granger Cause LN_BANKRATE	184	1.64128	0.1966
LN_BANKRATE does not Granger Cause D_SAVING		0.73762	0.4797
LN_PRIME_RATE does not Granger Cause LN_CONSOLS	255	2.68716	0.0700
LN_CONSOLS does not Granger Cause LN_PRIME_RATE		1.09302	0.3368
SAVING does not Granger Cause LN_CONSOLS	185	2.60227	0.0769
LN_CONSOLS does not Granger Cause SAVING		1.66107	0.1928
D_SAVING does not Granger Cause LN_CONSOLS	184	1.22549	0.2961
LN_CONSOLS does not Granger Cause D_SAVING		1.65606	0.1938
SAVING does not Granger Cause LN_PRIME_RATE	185	3.78845	**0.0245**
LN_PRIME_RATE does not Granger Cause SAVING		1.06493	0.3469
D_SAVING does not Granger Cause LN_PRIME_RATE	184	1.17080	0.3125
LN_PRIME_RATE does not Granger Cause D_SAVING		1.25035	0.2889
D_SAVING does not Granger Cause SAVING	184	NA	NA
SAVING does not Granger Cause D_SAVING		NA	NA

Annex 2. Unit root tests

ADF and PP unit root tests

Null Hypothesis: cariable has a unit root

	Augm. Dickey-Fuller		Phillips-Perron	
	t-Statistic	*Prob.*	*t-Statistic*	*Prob.*
ln_gdp	3.270726	1.00000	4.191411	1.00000
d(ln_gdp)	-15.6032	0.00000	-21.3507	0.00000
ln money	1.955354	0.99990	2.374669	1.00000
d(ln_money)	-14.45021	0.00000	-19.0968	0.00000
ln_deflator	2.709466	1.00000	2.837255	1.00000
d(ln_deflator)	-9.510791	0.00000	-10.0553	0.00000
ln_consol	-2.006722	0.28390	-1.91446	0.32540
d(ln_consol)	-11.81589	0.00000	-15.3501	0.00000
ln_saving	-4.66992	0.00010	-4.65138	0.00020
d(ln_saving)	-12.3296	0.00000	-15.2116	0.00000

Group unit root test: Summary **Levels**
Series: LN_GDP, LN_MONEY, LN_DEFLATOR, LN_BANK_RATE, LN_PRIME_COMMERCIAL, LN_CONSOLS, LN_SAVING, D_SAVING,
Sample: 1688 2016
Exogenous variables: Individual effects
Automatic selection of maximum lags
Automatic lag length selection based on SIC: 0 to 2
Newey-West automatic bandwidth selection and Bartlett kernel

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	6.46908	1.0000	9	2542
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-2.89973	0.0019	9	2542
ADF - Fisher Chi-square	146.103	0.0000	9	2542
PP - Fisher Chi-square	163.551	0.0000	9	2550

Group unit root test: Summary **First difference**
Exogenous variables: Individual effects
Automatic selection of maximum lags
Automatic lag length selection based on SIC: 0 to 7
Newey-West automatic bandwidth selection and Bartlett kernel

Method	Statistic	Prob.**	Cross-sections	Obs
Null: Unit root (assumes common unit root process)				
Levin, Lin & Chu t*	-31.6749	0.0000	9	2523
Null: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	-38.8902	0.0000	9	2523
ADF - Fisher Chi-square	909.323	0.0000	9	2523
PP - Fisher Chi-square	1032.56	0.0000	9	2541

** Probabilities for Fisher tests are computed using an asymptotic Chi -square distribution. All other tests assume asymptotic normality.

Table 3. Bounds test

Null hypothesis: No levels relationship
Number of cointegrating variables: 6
Trend type: Rest. constant (Case 2)
Sample size: 185

Test Statistic	Value
F-statistic	15.132169

	10%		5%		1%	
Sample Size	I(0)	I(1)	I(0)	I(1)	I(0)	I(1)
Asymptotic	1.990	2.940	2.270	3.280	2.880	3.990

* I(0) and I(1) are respectively the stationary and non-stationary bounds.

Table 4. Symmetry test

Coefficient symmetry tests
Null hypothesis: Coefficient is symmetric
Degrees of freedom (simple tests): F(1,167), Chi-square(1)
Degrees of freedom (joint tests): F(2,167), Chi-square(2)
Equation:

Variable	Statistic	Value	Probability
	Long-run		
LN_CONSOLS	F-statistic	37.06235	0.0000
	Chi-square	37.06235	0.0000
LN_DEFLATOR	F-statistic	0.994218	0.3202
	Chi-square	0.994218	0.3187
	Short-run		
LN_CONSOLS	F-statistic	7.538378	0.0067
	Chi-square	7.538378	0.0060
	Joint (Long-Run and Short-Run)		
LN_CONSOLS	F-statistic	18.60766	0.0000
	Chi-square	37.21532	0.0000

Annex 5. Estimated model

Dependent Variable: D(LN_GDP)
Method: ARDL
Sample: 1832 2016
Included observations: 185
Dependent lags: 4 (Automatic)
Automatic-lag linear regressors (4 max. lags): LN_MONEY LN_SAVING
Automatic-lag dual non-linear regressors (4 max. lags): LN_CONSOLS
Automatic-lag long-run non-linear regressors (4 max. lags): LN_DEFLATOR
Static regressors: DUM_WW
Deterministics: Restricted constant and no trend (Case 2)
Model selection method: Akaike info criterion (AIC)
Number of models evaluated: 2500
Selected model: ARDL(1,1,2,2,2)

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
LN_GDP(-1)	-0.164013	0.022997	-7.13197	0.0000
LN_MONEY(-1)	0.015869	0.014384	1.103235	0.2715
LN_SAVING(-1)	0.056294	0.007032	8.005602	0.0000
@CUMDP(LN_CONSOLS(-1))	0.047775	0.013906	3.435555	0.0007
@CUMDN(LN_CONSOLS(-1))	-0.061541	0.013625	-4.5169	0.0000
@CUMDP(LN_DEFLATOR(-1))	-0.021244	0.011595	-1.83212	0.0687
@CUMDN(LN_DEFLATOR(-1))	-0.004423	0.011855	-0.37306	0.7096
C	1.058811	0.176671	5.993108	0.0000
D(LN_MONEY)	0.106088	0.037971	2.793907	0.0058
D(LN_SAVING)	0.0358	0.008333	4.296373	0.0000
D(LN_SAVING(-1))	-0.026934	0.007295	-3.69185	0.0003
D(LN_DEFLATOR)	0.137601	0.058513	2.351621	0.0199
D(LN_DEFLATOR(-1))	-0.105735	0.050354	-2.09981	0.0372
@DCUMDP(LN_CONSOLS)	-0.042414	0.043265	-0.98032	0.3283
@DCUMDN(LN_CONSOLS)	0.003186	0.037626	0.084671	0.9326
@DCUMDP(LN_CONSOLS(-1))	-0.169171	0.045671	-3.70417	0.0003
@DCUMDN(LN_CONSOLS(-1))	0.028178	0.041947	0.671748	0.5027
DUM_WW	0.071092	0.011651	6.101626	0.0000

R-squared	0.463207	Mean dependent	0.013225
Adjusted R-squared	0.408563	S.D. dependent	0.028563
S.E. of regression	0.021967	Akaike info	-4.70636
Sum squared resid	0.080582	Schwarz criterion	-4.39303
Log likelihood	453.3382	Hannan-Quinn	-4.57937
F-statistic	8.476876	Durbin-Watson	1.952645
Prob(F-statistic)	0.00000		

*Note: p-values and any subsequent test results do not account for model selection.

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