Deliberation and Stochastic Consensus

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Theories of Deliberative Democracy are en vogue. They seem to give a new foundation and justification for democracy that goes beyond the procedural interpretation of liberal democracy. For theorists of European integration it has a particular attraction: it links the peace project, which is at the core of European integration, to institutional forms of political decision making. For as long as people (and governments) talk to each other, they will not shoot at each other. In recent years these theories have influenced academic debate on institutional arrangements, particularly with respect to a European Constitution (Eriksen and Fossum, 2000)

Theories of deliberative democracy have been inspired by Habermas' work on communicative action and discourse theory. One of its strongest points remains the shift from strategic to communicative rationality. People act strategically, when they chose a course of action that allows them the realization of their best preferences. Game theory models the complexities of strategic interaction of independent actors *with given sets of preferences*. However, preferences also change. The question is: why? Theories of deliberative democracy emphasize the epistemic quality of deliberation: in free, open and fair debates people will advance arguments for and against certain normative claims[2]. This will make a particular course of action more coherent and justifiable within a given normative context. This will render them legitimate. Hence, it will be preferable over another. In other words, preferences change for epistemic reasons in the process of communication. If democracy is to be based on such communicative rationality, clearly, it requires smart people capable of making judgments, conditions of free and fair speech and broad participation of citizens in the process of deliberation (a large public sphere). For otherwise the acceptance of a normative claim is not widely shared, and actions derived from this claim may lack legitimacy. From this point of view, deliberative democracy is a theory of epistemic consensus.

A number of objections have been raised. Empirically, the ideal conditions of free, open and fair debate are rarely fulfilled (Sanders, 1997; Knight and Johnson, 1994; Sunstein, 2004). Theoretically, one also encounters a paradox (Eriksen, 2007): if collective preferences are the *result* of a deliberation process, we need to assume the normative conditions of deliberation as given and justify preferences by the process through which they are formed. But then the theory of deliberative democracy is reduced to a procedural theory, similar to liberal democracy. Alternatively, if deliberative democracy is defended on grounds of the normative qualities of the deliberation process (free, open, fair), then it imposes *a priori* normative standards that are independent of deliberation. In this case the epistemic foundation of preferences does no longer depend on deliberation, but on the prior context of morality. Habermas has tried to solve the paradox by making deliberation dependent on context, but this privileges the normative substantialism of deliberative democracy. Eriksen seeks to avoid the high normative

[1] I am indebted to Majid Al-Sadoon, who helped to develop and formally prove several of the claims made in this paper.

[2] All courses of action are based on normative claims. The intentional *is* normative. Even the strategic rationality of standard utilitarianism in neoclassical economics implies the normative claim of optimizing utility given certain constraints.

standards of deliberative consensus theory by introducing the concept of "working agreement": "an outcome might fall short of a rational consensus but still be the result of a deliberative process based on inter-subjectively justifiable reasons. In line with this, one may think of the possibility of reaching an in-between consensus, an agreement which testifies to some movements of positions and normative learning, which does not result in rational consensus, but in a *working agreement*" (2006:16-17). This concept then stands between rational consensus and strategic compromise. Rational consensus is a form of unanimity, which occurs when actors can accept the validity of a claim for the *same* reasons. Rawls has softened this condition in his concept of *overlapping consensus*, where people may accept the validity of claims for different reason. By contrast, working agreements are based on *reasonable reasons*, which reflect the doubts and epistemic uncertainties, the bounded rationality, by which actors are confronted when they make decisions. Eriksen rightly claims that this is different from what Habermas calls an *understanding* (*Verständigung*). This situation describes an actor who is able to see that, with given preferences, another actor may have good reasons to act in a particular way *without being willing to make the reasons of the other his own*. Understandings in this sense will never lead to agreement or consensus (*Einverständnis*). Yet, as Lehrer and Wagner (1981) have shown, taking on another person's view for one's own assessment of what is right or desirable, for whatever reason, is a necessary and sufficient condition for consensus to emerge.[3] The question is therefore, how people come to accept another person's view and how this is compatible with people's differing capacities for reasoning. The question is not, how working agreement can be derived from understandings, but rather, how to reconcile the diversity of opinions and preferences that characterizes working agreements with the phenomenon of rational consensus, which leads to unanimity. In response, I propose a theory of stochastic consensus.

1. Building blocs of a theory of stochastic consensus

What do we mean when we say that there is consensus in a group? What is happening when people hold the same opinion, prefer the same thing? How is it possible for individuals to compare their personal beliefs and preferences and to say they are the same?

a. Mind states, beliefs, preferences and actions

When individuals have an opinion, a belief, a desire or a preference, they experience forms of consciousness that we call intentional mind states (Searle, 1979; 1983). Intentional mind states are experienced by a person, i.e. they are ontologically dependent on the existence of that person, and they are directed at an object. The object is what the mind state is about. It can be an actual or an imaginary state of the world, but either way it is a potential state of the world, which I will call an *option*. For example, I can wish to drive your car or to live in a world without war. These are different options for which I may experience a sensation of want.

An intentional mind state is the event of an individual having a thought about a potential state of the world. Therefore a tension exists between how the world is and how

[3] This formulation is deliberately vague. More precise definitions follow below.

it ought to be. Solving the tension between the two requires an action. Logically, desires and preferences imply an action, or sequence of actions, designed to make the world fit the intentions of the desiring person. However, it is important to emphasize that this is a logical and not a practical implication. It is possible to have a desire and not act on it. In fact, it is precisely this possible gap, which makes the difference between naturalistic desires and rational preferences, as we will see. When no action is required, the mind state and the world are in equilibrium. A true belief is such an equilibrium; a satisfied desire as well. The difference between beliefs and desires or preferences consists in the direction of fit: when reality does not correspond to the intentional mind state, I may adjust my *belief* in order to make it "true". Alternatively, if I have a *desire*, I should adjust reality to make it fit my intentional mind state. Thus, beliefs have a world-to-mind direction of fit, desires and preferences have a mind-to-world direction of fit. The theory of stochastic consensus applies to both cases, but in the following I will concentrate on desires and preferences, as they are more relevant to collective decision making. I define a desire as the event of an individual experiencing the sensation of desiring, wishing, wanting etc. with respect to an option. A preference is a desire, which has been reflected upon by the individual that experiences the sensation.

Sensations are specific to individuals and their experience cannot be shared with anyone else. Economists have argued that comparing utilities (intensities of preferences) is impossible, because "we cannot look into each other's heads" (Elster and Romer, 1991). Therefore, we also cannot know directly what motivates another person other than what her action reveals (Samuelson, 1947). Yet, people actually make comparisons of well-being all the time, because they talk to each other ("how are you?"). They speak and listen, they understand or fail to do so and they agree or disagree with each other. They communicate in private and public life, in markets and hierarchical institutions, as private agents or collectively as groups and organizations. Communication is necessary not only for interpersonal comparisons, but for the sharing of any kind of mental state and cooperative behaviour. However, communication is not unambiguous. People do not always represent the world as it is. They can misperceive and err; they may pretend or lie. In order to focus on how people come to accept another person's view, we will assume that mind states can be expressed by speech acts and that individuals are communicating sincerely and truthfully, so that making a statement is equivalent to having a corresponding mind state. Whether a person has a desire can then be re-framed in terms of whether he/she *accepts* a statement as the proper description of his/her mind state.

b. Intensities of desire and context

Given that mind states are contained in certain states of the world, we can model their occurrence as random events and their sequence as a random process. This means mind states evolve as a stochastic process whose repeated realizations follow no describable deterministic pattern, but a probability distribution. This idea may at first surprise, but with little introspection anybody will acknowledge that specific thoughts occur in an unpredictable flow of mind states where certain options appear more frequently than others.

The concept of frequency of random events allows us to interpret the probability of a specific desire occurring as the intensity of that desire. If I keep thinking repeatedly of wanting to eat chocolate, my desire for eating chocolate is high. If I rarely think of it, it is low. Similarly, if I frequently think that a proposition is true and rarely doubt it, I have a strong belief. This interpretation allows reformulating the classical utility function. Preferences are not a *mapping of a set of options* into the set of real numbers, or even into the interval [0.1], but they are *mapping mind states* about a set of options into the interval [1,0].[4] Thus, I propose a probabilistic interpretation of utility and preferences. Because it shifts the focus on mind states away from options, this approach can be used to overcome the rigidity of exogenous preferences and to endogenize their evolution.[5]

Even if mind states are random events, they do not occur without context. Context may be anything that contributes to the emergence of thoughts, whether natural, biological or cultural. When it is hot, I may have a stronger desire for a cool drink than when it is cold. When I buy a house, my preference for low interest rates is likely to be higher, than when I live on my investments. Context is what is there prior but not unrelated to the emerging thought. It is the *Living World* in Habermas' sense that is "always already there". Therefore specific thoughts are conditional on the context in which they occur.

Technically, this implies that the intensity of a desire is represented by the conditional probability in a given and specifiable context. Desires, which arise from a given context without further reflection by the individual, I will call *naturalistic* preferences. They appear as natural as stimulus-response behaviour of any animal, even if the conditioning context is cultural habit and not biological.

c. *Preference ranking and agreement*

Modeling intensities of preferences as conditional probabilities implies that they can be compared across options and across individuals. For example, given the context of hot weather and a limited choice of beverages, I may have two options, drinking cold water or hot tea. If the probability of the thought "I want to drink water" is higher than that of "I want to drink tea", then we say water is preferred over tea. The logical implication for action would be to go and drink water. However, notice that the preference order is only a necessary, not a sufficient condition for action. As Sen has pointed out, in order to get choice the preference ranking must be complemented by a decision rule. The decision rule transforms the logical implication for action implied by the preference into actual action for making the world fit the intention.

In addition to intra-personal preference ranking, inter-personal comparisons are also possible because people can express their feelings and communicate with each other. In principle it is therefore possible to describe the conditions of communication that allow a

[4] Conventional utility functions are described as $u : X \rightarrow R$ (where X is the option set and R the set of real numbers), while our utility function should be described as $u : M(X) \rightarrow [0,1]$, where M(X) is an intentional mind state directed at the option set X.

[5] Because neo-classical utility theory assigns utility values to options (commodities), utility can only vary with quantities of commodities, for example by assuming a law of diminishing marginal utilities. I would not dispute this law, but would emphasize that the utility function would change with changes in mind states that result from deliberation.

correct description of preference intensities. This allows us to make a subtle distinction between consensus and agreement. *Consensus* occurs between individuals when they have the same intensity of preference, i.e. the same probability of accepting the mind state as worthy of action. All members of a group are then equally likely to accept or reject a statement of desire or preference. *Agreement* signifies a state of the world where all individuals have the same preference ranking, but not the same intensity of preferences. For example you may desire to drink water with a high intensity of 70 % and tea only with 30%. I may be less extreme, but still prefer water over tea with a probability 60:40. Hence we agree on the preference ranking, although we do not have consensus over our preferences. Thus, our notion of agreement over preference ranking comes close to Eriksen's (2006) concept of working agreements.

2. Deliberation

Rational beings have the capacity to evaluate their desires. The may consider different aspects of what it means to follow the course of action that is logically implied by their desire: What are the reasons, what are the consequences? Furthermore, individuals may be aware that they alone may not be able to judge what is appropriate. They therefore may look at the opinions of others and, as we will see, this will lead to consensus among the individuals of a group.

a. Rational deliberation

Even though naturalistic preferences are determined by their context, individuals may deliberately broaden the context and evaluate the intensity of their desire in view of additional arguments. Even if I have a strong desire to eat chocolate when I walk by a chocolate shop, I may reflect on the desirability and the consequences implied by my desire for my health, weight, social approval, etc., and this may affect the intensity of my desire. I will call a "revalued desire" a *rational preference*. Evaluating a desire implies accepting or rejecting this desire as a justification for action. The intensity of a rational preference therefore reflects the initial intensity of a desire *and* the likelihood that this desire has been retained or accepted as the basis for action. The acceptance or rejection of a desire as worthy for action is based on arguments, which justify this retention. The strength of arguments can also be expressed and measured in probabilistic terms. As a consequence, the variation of preference intensity, given the strength of a new argument, can be modeled as a Bayesian updating process. Without going into the formal details, we retain here that the modification of naturalistic desires depends on the reliability of an argument and its innovation over the existing informational background in the process of evaluation.[6]

[6] The Bayesian updating is formally described as:

$\pi = p(s \mid C \& E) = \frac{p(E \mid s \& C)}{p(E \mid C)} p(s \mid C)$, where π is the preference intensity, s stands for the desiring mind state, E is the evidence of a new argument and C is the context. p(s|C) is the naturalistic

This process of rational deliberation leads to a *reflective equilibrium* that is somehow shared by fully rational individuals (Rawls, 1987; Habermas, 1981; Kolm, 2005). Each individual ponders the weight of arguments and adjusts his or her personal preference intensities. This is what most of the literature describes as the conditions of deliberative democracy. For reasons which are never made explicit, it is assumed that the open nature of the public debate among free and equal individuals produces an epistemic consensus on the weight of arguments and therefore rational individual should agree. Consensus is then a consequence of convincing reasons (Bohman, 1996:25). But from a Bayesian point of view, this is not a necessary consequence. For even if you and I gave the same weight to all arguments and we had discussed the issue long enough to have exhausted all aspects of novelty, our posterior probabilities could still diverge if our prior preference intensities had been at variance. This is the problem with *understandings*, as discussed before. Discourse theory compounds the problem of explaining how consensus may come about by the fact that opinions and preferences are always described in exclusively qualitative terms. It does not attach a quantitative measure of preference intensity to these qualitative descriptions. But in terms of stochastic consensus theory, different qualitative statements are different options. What needs explanation is *not that* people switch from one preferred option, but *how come* they do so? Discourse theory does not provide an answer and therefore gets caught in the paradox described above.

b. Deliberation with bounded rationality

The theory of stochastic consensus provides a simple, powerful reason why individuals re-evaluate their preference intensities and converge to consensus: bounded rationality. It builds on previous work by deGroot (1974) and Lehrer and Wagner (1981), who interpreted consensus as optimal opinion pools. Bounded rationality assumes that individuals are aware of their limited capacity to process all relevant information in forming views on preferences. They therefore look to other people for advice, help and the correction of their misperceptions. They exchange ideas, arguments, reasons, judgments and opinions. They convey their own thoughts to others and learn from the opinion of others. This permanently ongoing process of communication transforms individual preferences into social preferences, which ultimately *must* converge to consensus, provided two simple conditions hold. We now analyze this process.

To keep things simple, we assume that the time process progresses in discrete steps. In the initial stage, all individuals have worked out their own rational preference over a range of given options. But even if they have done so to the best of their abilities, they are aware that other arguments may exist, which, although they may not know and understand them, could lead them to change the probability assessment of their own rational preference. In other words, they doubt their own judgments. In order to improve the information set of their preference assessment, they look at the assessments of other people they respect. Consequently, the probability that they accept an option as their preference will change. If they respect some other people strongly, they will in return be

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preference as determined by the context, p(E|C) is the innovation effect and p(E|s&C) measures the reliability of the argument. See Collignon, 2003, annex I.

strongly influenced by them. Their preference intensities will become similar to those whom they trust. If they do not respect them, they keep their own preference intensity. Thus, rational behavior under the constraint of limited cognitive capacities requires that each person re-assess his/her preference intensity as the weighted average of all persons he or she respects.

The question is now: what determines the weights that individuals assign to others? Lehrer and Wagner (1981) use the heuristic trick of assuming that each person splits and attributes a unit-weight over all persons he or she respects, so that the weights are a measure of relative respect. In doing so, they express some subjective probability distribution for the option, but the factors, which determine each share, remain unexplained.

A different way of interpreting these weights is to see them as transition probabilities: they describe the probability that I will move from my old position to the preference intensity of the people I trust. The reasons for the shift are not specified, although rational arguments, recognition and personal reputation all matter prominently. In a given group each individual influences potentially any other, provided (s)he gains respect from other persons. If I include the self-respect I have for myself, the probability that I will end up in the next period either at my old position or at someone else's position is equal to one (i.e. it is a certain event). Thus, each individual's transit probability is the weight that he/she effectively attaches to another person's preference intensity.[7]

For example, take the simplest case of two individuals. The transition matrix W looks like this:

$$(1) \qquad \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} = W$$

In the first row α represents the probability that person *A* will move from his own original preference intensity to that of person *B*. $1-\alpha$ is then the probability that he keeps his original view. Similarly, in row two, β represents the respect *B* grants to *A* and $1-\beta$ for herself. The probabilities in each row add up to 1, i.e. in the new period each person must have adopted either the view of another person or remained on her previous position. A matrix where the elements of each row add to one is called a stochastic matrix. If we use it to describe large groups or even society, it will contain many cells, where the entry is zero. In fact, most people will at first only trust their immediate environment.

Interpreting the weights as transition probabilities has two advantages: first, it allows synthesizing the knowledge of different people into a single optimal information set. It allows aggregating preferences of different people without having to impose ideal speech act or discourse conditions. The interpretation also is indifferent to the moral standards of deliberation or to the status of individuals. Some may be very influential, others hardly at all. Consensus will emerge, even if people are not treated equally and have very different

[7] There is some formal resemblance to the Lehrer and Wagner approach in this assignment, but in substance our transition probabilities are no longer a subjective probability assignment to option, but an objective description of the likelihood that a person will switch his opinion.

intellectual capacities.[8] This model, therefore, avoids the paradox of deliberative democracy theories, mentioned in the introduction.

Secondly, the transition probability can be interpreted from the point of view of the person *who gives respect* or from the point of view of the person *who influences* those who ask for advice. If we describe a society in the form of a square matrix, where each line represents one person's respect to all others (including himself), i.e. the transition probabilities from all previous preference assessments to his assessment today, then we find the influence of a person represented in the column vector of the matrix. The fuller the column[9] and the higher the weights in a person's column, the more influential is this individual. Thus, the structure of the weight matrix will depend on the outcome of the ongoing "struggle for recognition", i.e. on the competing claims of individuals for the respect and trust of others.

c. Convergence to unanimity

If the weight matrix *W* gives the transition probabilities that an individual will adopt a different view in the new period then a person will maximize the informational content of his deliberation by taking the weighted average over people's preference intensities. This is not a normative claim but a logical implication of the assumption of bounded rationality: If I cannot judge correctly on my own, I must listen to those who I believe know better, if I am to behave rationally. Thus, I adjust my own intensity of desire.

For example in the above mentioned example, person *A* will adjust the probability for accepting an option as his preference to:

(2) $$\pi_A^1 = (1-\alpha)\pi_A^0 + \alpha\pi_B^0,$$

where π_A^0 is the probability that *A* accepts the option in period 0. *B* does the equivalent updating. In more general terms, we can write this as:

(3) $$\Pi^1 = W\Pi^0,$$

where Π is the matrix of preference intensities with rows listing the individuals and columns the options.

Thus the new distribution of preference intensities has changed over the initial distribution. However, the fact that a person has changed his assessment contains information for all others who respect him. Therefore a second round of deliberation takes place that leads to a further adjustment of individual preferences. And after that, a third one, etc. This process of deliberation will go on until all individuals have the same distribution of preference intensities, that is, until they will all accept an option as their preference with the same probability. At that point unanimous consensus is achieved. However, this unanimity is very different from the deterministic consensus concept that dominates economics, particularly in the Pareto-optimality theorem or in Buchanan and Tullock's (1962) calculus of consent. Rather than excluding all other options from consensus, stochastic consensus includes them in the probability distribution. It therefore describes a pluralistic form of consensus.

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[8] The equality postulate was first formulated by Harsanyi (1955). For a critique see Lehrer and Wagner, 1981.

[9] Political opinion polls on the popularity of politicians often ask whether a certain personality is known to the interviewed person. For highly visible people, such as a country's president, the weight matrix has no empty cells in the column.

In formal terms the second round looks like this: $\Pi^2 = W\Pi^1 = WW\Pi^0 = W^2\Pi^0$. And after repeated steps of deliberation:

(4) $\Pi^t = W^t\Pi^0$

If we keep going with infinite repetitions of deliberation, i.e. $\lim t \to \infty$, we get the unanimous consensus distribution as:

(5) $\bar{\Pi} = W^\infty\Pi^0 = \bar{W}\Pi^0$

This is an interesting result. We know that as the power of multiplying stochastic matrices becomes large (theoretically infinity), the weight matrix converges toward a stationary distribution where every row has exactly the same value. In other words, each individual gives exactly the same value of respect to any specific individual that any other member of the community gives him too. There is a consensus in society about whom one should trust how much with respect to evaluating a specific issue. And because of the rationality assumption, everyone takes the weighted average of theses people's opinions, which results in everyone having the same probability distribution of preferences.[10]

Note also that an individual may start out in the deliberative process thinking that he has no capacity to judge whether an option is desirable (his cell in the weight matrix W is zero). But after several rounds of arguments he will discover that others have some respect for his original view. Because he trusts those others, he will start trusting himself. At that point his cell in the power weight matrix W^t is no longer zero and we say the person eventually has self-regard.

We find that social structures between individuals produce consensus in preferences, while the intellectual openness for and the integration of new evidence will shift rational preferences. If we want to understand, how consensus on policy issues comes about, we need to study the structure of the weight matrix.

3. Communication

This analysis raises two questions. First, under what conditions is convergence to unanimity assured? Second, how long does it take, until it is achieved? This first question is well established in the literature (see deGroot, 1974; Lehrer and Wagner, 1981), the second is a frontier for research.

a. Consensus, dissent and conflict

Under the assumption of bounded rationality, individuals are not totally convinced that they know all there is to know about a subject and therefore will communicate with others in order to improve their preference assessment. When individuals communicate,

[10] In other words, social structures of communication affect the weight matrix *W*, while new evidence on the subject matter affects Π^0.

they respect and influence each other. Two fundamental conditions are necessary and sufficient for unanimous consensus to emerge:[11]

Condition 1: *connectedness*. All individuals of the group need to be connected, even if only indirectly.[12] The connectedness condition requires that there is a path from any individual *m* to any individual *n* through which any two individuals influence each other.[13] This condition ensures the inclusiveness of consensus.

Condition 2: *minimal self-regard*. At least one person in the group must eventually have self-regard, for if no person ever takes on her own view, deliberation will just pass arguments on from one person to the next without anybody accepting them as their own. Minimum self-regard is required to prevent deliberation from infinite cycling.

If these two conditions are fulfilled, every individual influences every other and consensus will inevitably emerge. Consensus is represented by a row vector in the weight matrix of strictly positive elements that is identical for each individual.[14] In other words, in the equilibrium state of consensus, each individual gives the same weight as anyone else to each individual's prior rational preferences. Every member of society will therefore accept an option with the same probability as anyone else. Consensus in weight-giving implies consensus in preferences. Furthermore, because the limiting weights in the power weight matrix W^t are strictly less than one, consensus preference is non-dictatorial; no single person will impose his will on the group by influencing everyone else, without being influenced by others.

Perfectly unanimous consensus requires in theory infinitely long deliberation. In practice, this may be achieved much earlier, but the consensus toward which the distribution of preference intensities converges is a useful benchmark for the equilibrium position where all individuals would accept an option with the same probability. We can then use this equilibrium to measure the variance of individual preference intensities before unanimous consensus is achieved. I call this variance *dissent*. It is an indicator of how far we are from consensus.

Dissent can be measured by the average of squared deviations of the current set of preferences from the consensus preference assignment:[15]

$$(6) \qquad Diss(\pi^t) = \frac{1}{n}\sum_{j=1}^{n}(\pi_j^t - \pi^*)^2$$

It has a very precise meaning insofar as it implies that the two fundamental conditions of connectedness and bounded rationality hold, but the process of iterative deliberation has not yet gone to its end. Dissent is therefore higher at the beginning of the deliberative process than at a later stage. Dissent in this technical sense is compatible with agreement

[11] For simplicity I have assumed that the weight matrix does not change during the process of deliberation. This is not a necessary condition for consensus. Lehrer and Wagner have shown that consensus also occurs when the weight matrix changes.

[12] If *A* does not respect *C*, but only *B* and *B* respects *C*, all three of them are connected.

[13] The weight matrix is then called irreducible.

[14] This follows from the theory of Markov Chains, see Bremaud, 1999. The application to consensus is found in Lehrer and Wagner, 1981.

[15] Where π^t is the preference intensities vector at deliberation round *t* and π^* is the consensus preference vector. The group consists of n individuals.

on preference orderings, as we have seen above. However, until unanimous preference orderings are achieved deliberation may still take many rounds of re-iteration.

Dissent must be distinguished from *conflict*. Conflict occurs, when the fundamental condition of connectedness is violated. More precisely, when two (or more) individuals in the group are unwilling to give any respect to other people because they think they alone know what is best with respect to a given option, the group will split into two (or more) consensual camps with incompatible and non-converging preferences. This leaves two possibilities for dealing with conflict: brute force by eliminating the opponent or a meta-agreement on how to deal with conflict. For example, voting on specific issues, or more generally constitutional texts, are meta-agreements. However, these meta-agreements are necessarily themselves grounded in a constitutional consensus. Without such consensus, conflict will inevitably end up in violence and war.

b. Speed of convergence

How long does it take to converge to consensus and eliminate dissent? The answer is not straight forward, but a few general rules may be found.

Markov chain theory tells us that if conditions 1 and 2 are satisfied (so that the deliberation process can be described by the dynamics of Markov chains), dissent will geometrically converge to zero at the rate of $\lambda_2(W)$, the modulus of the subdominant eigenvalue of the weight matrix *W*. The closer $\lambda_2(W)$ is to 0 the faster the convergence to consensus; the closer it is to 1, the slower the convergence.[16] Unfortunately it is difficult to calculate the subdominant eigenvalue of large matrices. Even defining some upper bounds does not yield very significant results for determining how to accelerate the elimination of dissent. Nevertheless, a few simple observations can be made.

Given that consensus depends on communication, the length and intensity of the communication process must be important. One measure of length of communication in terms of numbers of deliberation steps required for the communication of the preferences between two individuals is the *degree of separation* in a society. If we take the shortest path of communication between any two individuals in the group, the degree of separation is the longest of these minimal communication paths. In other words, it describes how far apart the two most distant individuals in society are. On the other hand, the intensity of communication reflects the strength of influence one person has on another. The communication path with the weakest influence between to people can be thought of as the strength of the *weak ties* between individuals in a society.[17] Reducing the degree of separation in society or strengthening weak ties will contribute to faster elimination of dissent.

Given the difficulties of the mathematical *a priori* determination of the subdominant eigenvalue of *W*, we will recur to simulations of some structural models below. Before doing this, we must discuss some violations of the two basic conditions for stochastic consensus.

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[16] See Theorem 6.11 in Bremaud, 1990.

[17] The concept of *degree of separation* became popular through Stanley Milgram's (1967) "small world" experiments, which showed that people in the USA are on average connected by six degrees of separation. The idea of the *strength of the weakest tie* in society goes back to Granovetter, 1973.

c. The structure of communication

Theories of deliberative democracy emphasize the need for communication. The theory of stochastic consensus supports this claim, because it shows that efficient communication can contribute to the collective agreement on preference orderings. However, even under optimal conditions it may take longer to achieve consensus than what may be acceptable for efficient decision making.

Voting is a device to cut short the deliberation process. If the political system is based on universal suffrage and encourages broad and inclusive communication, intensive campaigning and inclusive public debate, election results will reflect the consensus distribution of equilibrium preference orderings is high.[18] Voting is then an approximation for finding the consensual preference ordering in society. This result also solves the famous voting paradox described by Condorcet, whereby voting on three options may end in a cycle. Arrow (1963) had taken this to formulate the incompatibility theorem, whereby aggregating of preferences under some simple assumption is incompatible with democracy (non-dictatorship). For many political scientists these theories seemed to imply a profound instability of democratic systems. In the context of stochastic consensus theory, the paradox applies to static preferences at a given moment, but it dissolves if the deliberation horizon is extended, so that dissent is reduced and ultimately even consensus achieved.

However, the success of this process largely depends on the structure of communication. Several violations of conditions 1 and 2 are possible.

Cyclicality occurs when condition 1 holds, but not condition 2. An example for infinite cycling is when two people try to go through a door, each politely asking the other to go first. Ultimately one person will have to accept to be first. With respect to deliberation, it implies that at least one person must accept a preference as his own. Otherwise, deliberation remains empty talk. Individuals would oscillate indefinitely between different preference-intensities.

Disconnectedness happens, when a society splits into factions. A collection of individuals will give regard to all members of their own group, but not to anyone else. The same is true for the other faction. Each group will then develop a group consensus, but preferences between the two factions will not converge. If the actions implied by the respective group consensus are incompatible, conflict will arise. Note, that because consensus within the faction implies that the weights in the row of the power weight matrix are identical for each group member, but zero for the members of all other groups, group consensus appears to be a form of group identity. Conflict between factions therefore seems to be motivated by group identities.

Dominance is a weaker form of disconnectedness. It implies that there is a dominant group, whose members let themselves only be influenced by other members of their own group, while there is a dominated group, which does respect at least some members of the dominating group without receiving any respect in return. In this case, society reaches a consensus that is completely determined by the dominant group. During the process of

[18] More precisely, if conditions 1 and 2 are satified, then (i) society will eventually reach agreement and (ii) the probability that the eventual agreement will be on accepting a specified preference is equal to the limiting probability of agreement on accepting the preference, which in turn is the same as consensus on the preference. For the mathematical proof see Collignon and Al-Sadoon, 2006, unpublished manuscript.

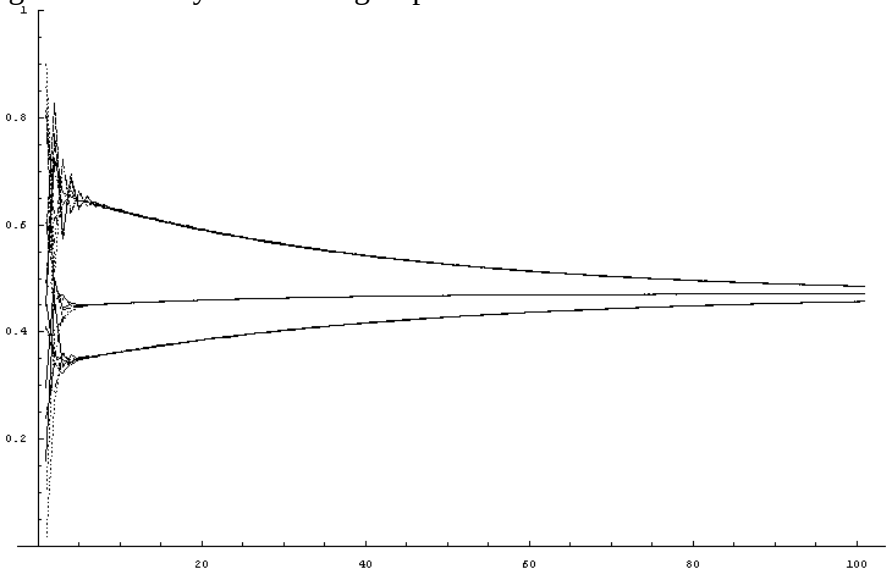
deliberation, the weights assigned to members of the dominated group go toward zero. Therefore, this form of consensus is non-inclusive. Examples are societies with cast systems, apartheid or ruling elites or oligarchies, where "ordinary" people are disregarded.

4. Some applications of the theory

a. Communitarianism

Communitarian groups emerge when groups are formed within society where individuals are densely connected within their groups but give little regard to the other groups.[19] This system was first studied by Simon and Ando (1961) under the notion of Nearly Completely Decomposable Systems. It implies that conditions 1 and 2 hold, but respect between different communities is weak. In this case consensus will emerge rapidly within groups, but only slowly between groups. Because conditions 1 and 2 hold, society will ultimately converge to unanimous consensus, but dissent is dominated by inter-group relations and not by noise of individual deliberation. Figure 1 shows[20] how preferences assignments converge quickly within the group and slowly between groups in a simulation based on Simon and Ando's systemic structure.

Figure 1. Loosely connected groups



Loosely connected groups explain the emergence of communitarian group identities, which take a long time to dissolve. For example within the European Union, policy consensus is slowly emerging, but even after half a century of integration national identities still seem to be the dominant feature. The convergence to consensus is accelerated by opening the community and increasing the regard individuals give to members of other communities. This can be done either by building trust between the

[19] See Emirbayer and Goodwin (1994)
[20] On the vertical axis we find the preference intensities, on the horizontal axis the number of deliberations.

communities (increasing the weights of non-members in the weight matrix), or by institutional features, which broaden the inclusiveness of deliberation.[21]

b. Constitutions of deliberative democracy

We are now interested to find out how constitutional structures may influence the speed of convergence to consensus. We will focus on some stylized facts, wwhich are of particular relevance for the European constitutional debate, but they apply *mutatis mutandis* to many kinds of organizational relations.

We assume that each system or society consists of two countries (groups) *A* and *B*. Country *A* consists of a government *GA* and *n* individuals; likewise country *B* consists of a government *GB* and *m* individuals; we also consider the effect of an additional player (an international organization). Each system or society consists of two countries (groups) *A* and *B*. Country *A* consists of a government *GA* and *n* individuals; likewise country *B* consists of a government *GB* and *m* individuals; we also consider the effect of an additional player (an international organization). We will ignore interactions among citizens of the same country and between citizens of different countries and focuses instead on international linkages and government-citizen linkages. Our purpose is to determine how institutional features affect the likelihood of converging more or less rapidly to consensus.

We compare three models of institutional interaction.[22]

The Intergovernmental Model (IG) assumes that governments influence each other and have self-regard. They also have regard for their citizens, while citizens only give regard to their own government and themselves. This model is similar to the structure of a two-level game (Putnam 1988), where different governments seek agreements among each other (level I) but are also dependent on acceptance by their constituency (level II).

The International Organization Model (IO) adds an international organization as a third player at level I. We call it "the Commission" and assume that it does not interact with citizens directly. The international organization model, therefore, has four additional links over the Intergovernmental model.

The Federal Republic Model (FR) models the international organization as a supranational government. Citizens are now influencing and being influenced by this supranational government, which gives equal regard to every individual of the enlarged constituency (m+n). However, the Federal Government also interacts with member state Governments.

Within the structure of the model infinitely many parameters will produce infinitely many different subdominant eigenvalues. We are interested in the distributions of these eigenvalues and wish to compare their distributions across the models. As there is no tractable formula for calculating them, we have simulated 10 000 parameters randomly after imposing the structure of our stylized facts and then run Kolmogorov-Smirnov test[23]

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[21] I have analyzed the role of cross-border trust building for the creation of monetary union in Collignon and Schwarzer, 2003; Collignon, 2003, focuses on institutional features for stronger group connections and broader and more inclusive deliberation.

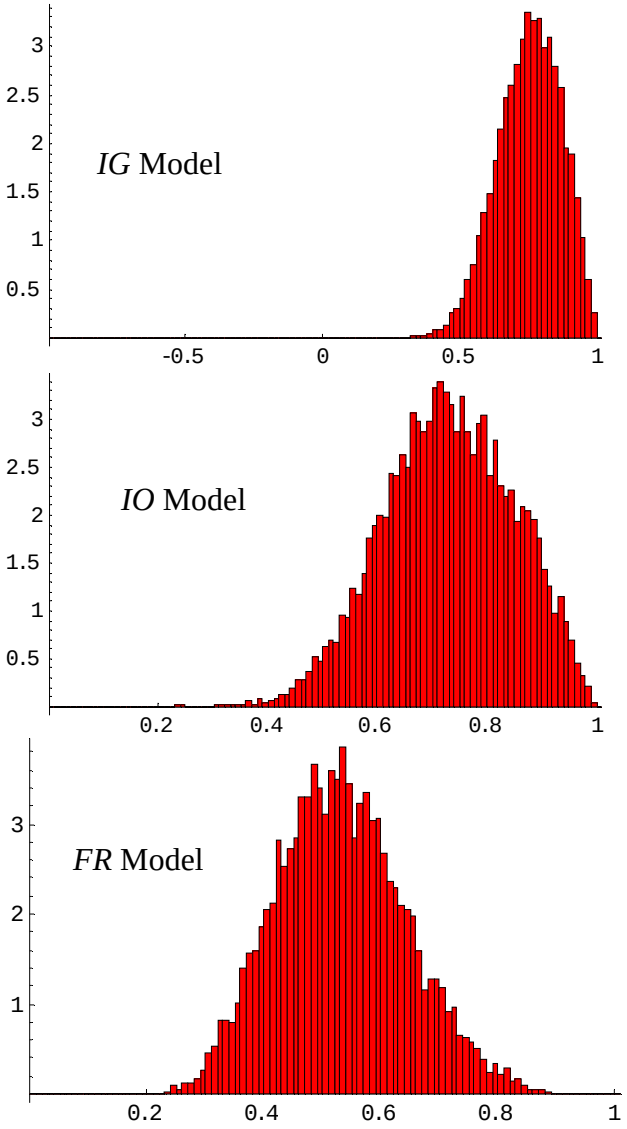
[22] The formal structure of each model's matrix is given in the annex.

[23] The Kolmogorov-Smirnov, or KS test, is used to test whether two distributions are identical against the alternative that they are not or against the alternative that one distribution is higher than the other or vice versa. They assess stochastic dominance in non-parametric statistics. When applied to the comparisons

to check if the distributions are significantly different of each other. Here are the results. A high concentration of values to the left indicate high speed of convergence.

Complete Ignorance. The first simulation draws the parameters in each row of the weight matrix from the same distribution. This makes sense for someone who knows the possible political structures but cannot ascertain the weights people give to each other or the proportion of people in each country. She may be behind a veil of ignorance, for example, trying to decide which political structure is best. Figure 2 shows the eigenvalue distributions in the three models.

Figure 2. The Complete Ignorance Simulation

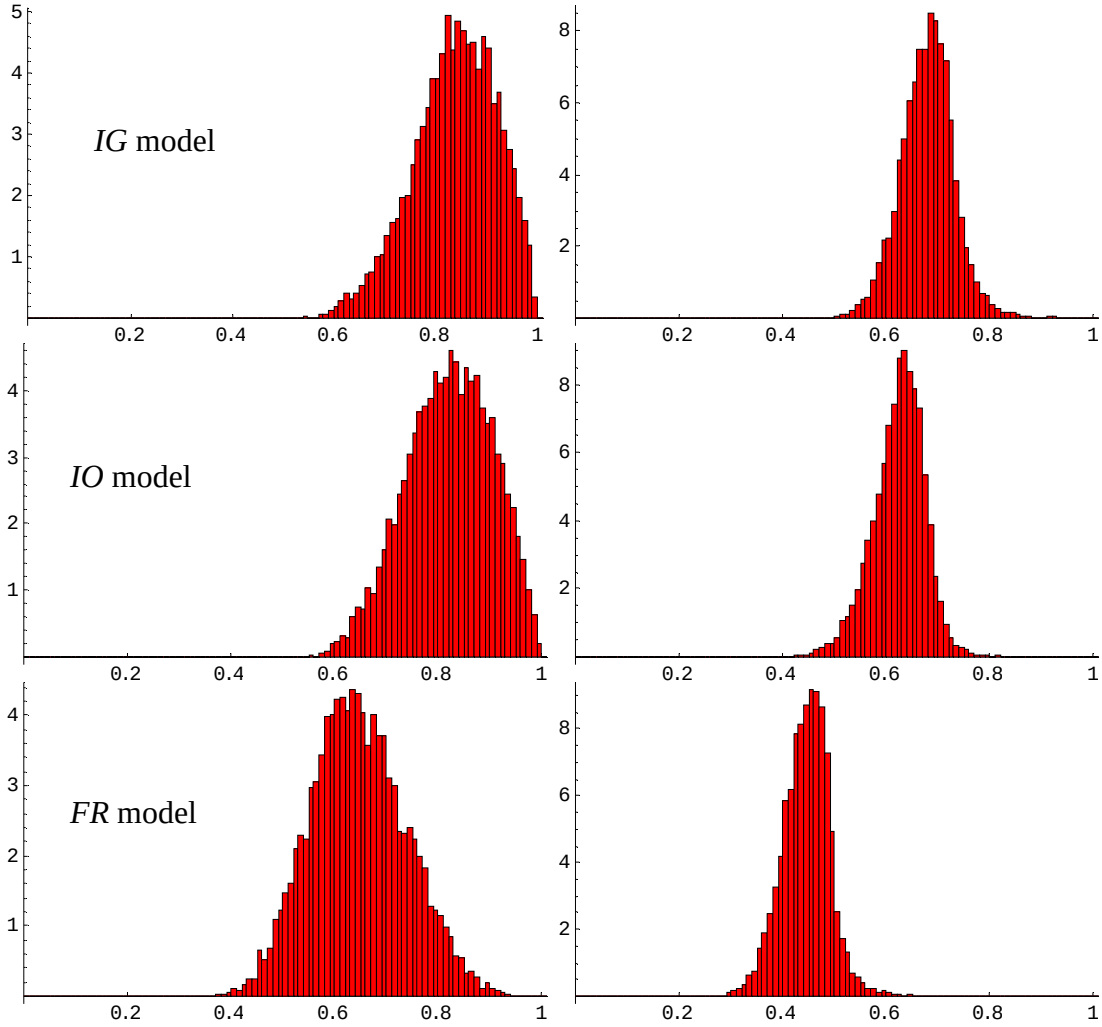


across the IG, IO and FR models, we will call them structural ranking tests. Test results are in Collignon and Al-Sadoon, 2006 and can be made available on request.

It is clear from the histograms, and confirmed by Kolmogorow-Smirnow (KS) tests, that the *FR* model is likely to converge the fastest, with the *IO* trailing next, and the *IG* model being the slowest of the three.

Pig-headedness and Open-mindedness. Next we consider two worlds. One is populated by pigheaded people (in both countries), who are strongly convinced of their own opinions and are unyielding to others; we model this by assuming that the value for each diagonal element in the weight matrix is larger than any other element in its row. The other world consists of open-minded people each of whom regards every other individual more than himself; here each diagonal element is smaller than every other element in its row.

Figure 3. The Pigheadedness vs. Open-mindedness

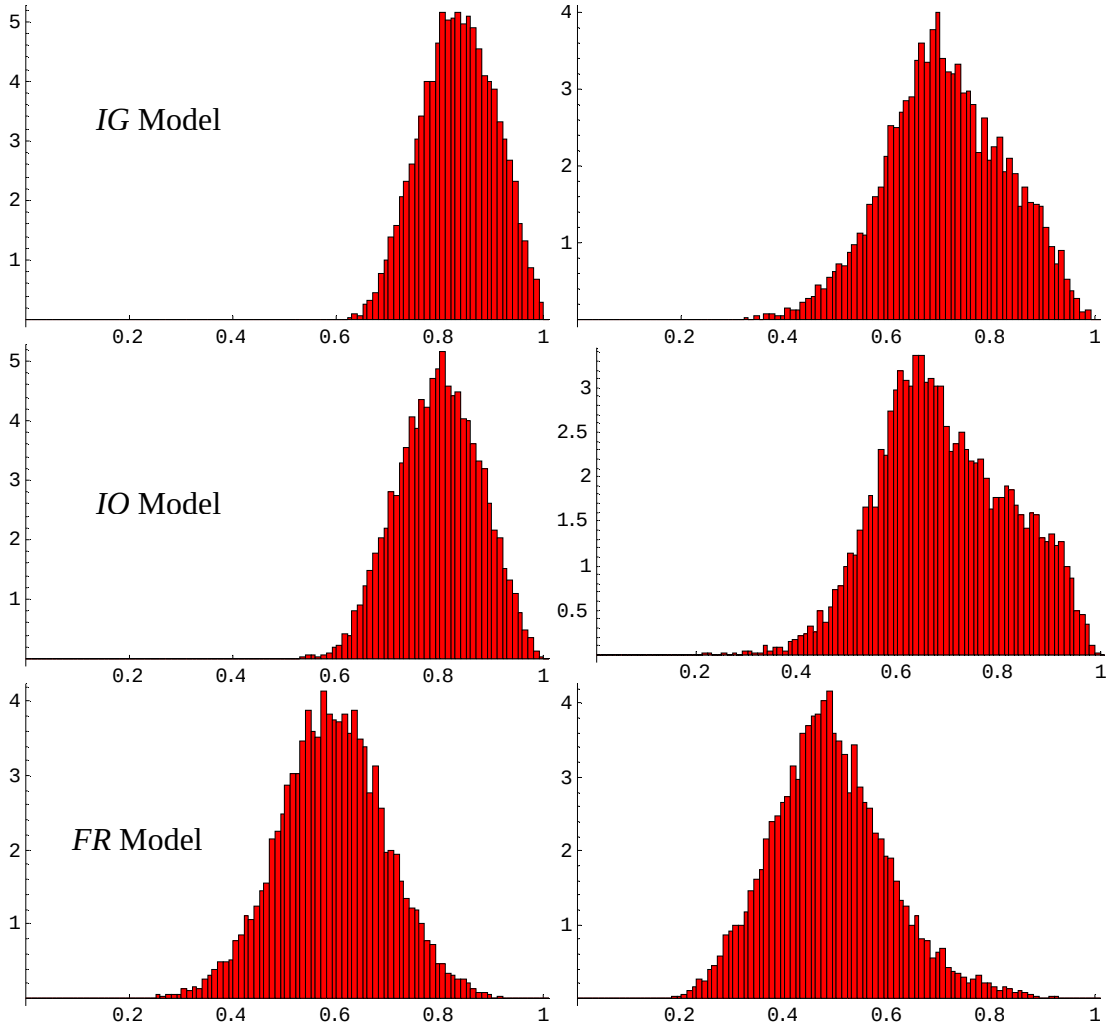


Pig-heads are so convinced of themselves that it is difficult to reach consensus; self-doubters converge faster than pig-heads across all three models. This result is confirmed by the KS test (at higher than 0.999 significance level) for the general distinction between the two attitudes, and for the three political models. The *FR* model is faster than the *IO*

model, which is faster than the *IG* model no matter how open-minded or pigheaded people are.

Liberal and Authoritarian Governments. We define a liberal world as one in which governments (including the Federal Republic) keep their citizens in relatively high regard. An authoritarian government is the exact opposite: each government gives less regard to its people than it does to other governments.

Figure 4. Liberal vs. Authoritarian Governments



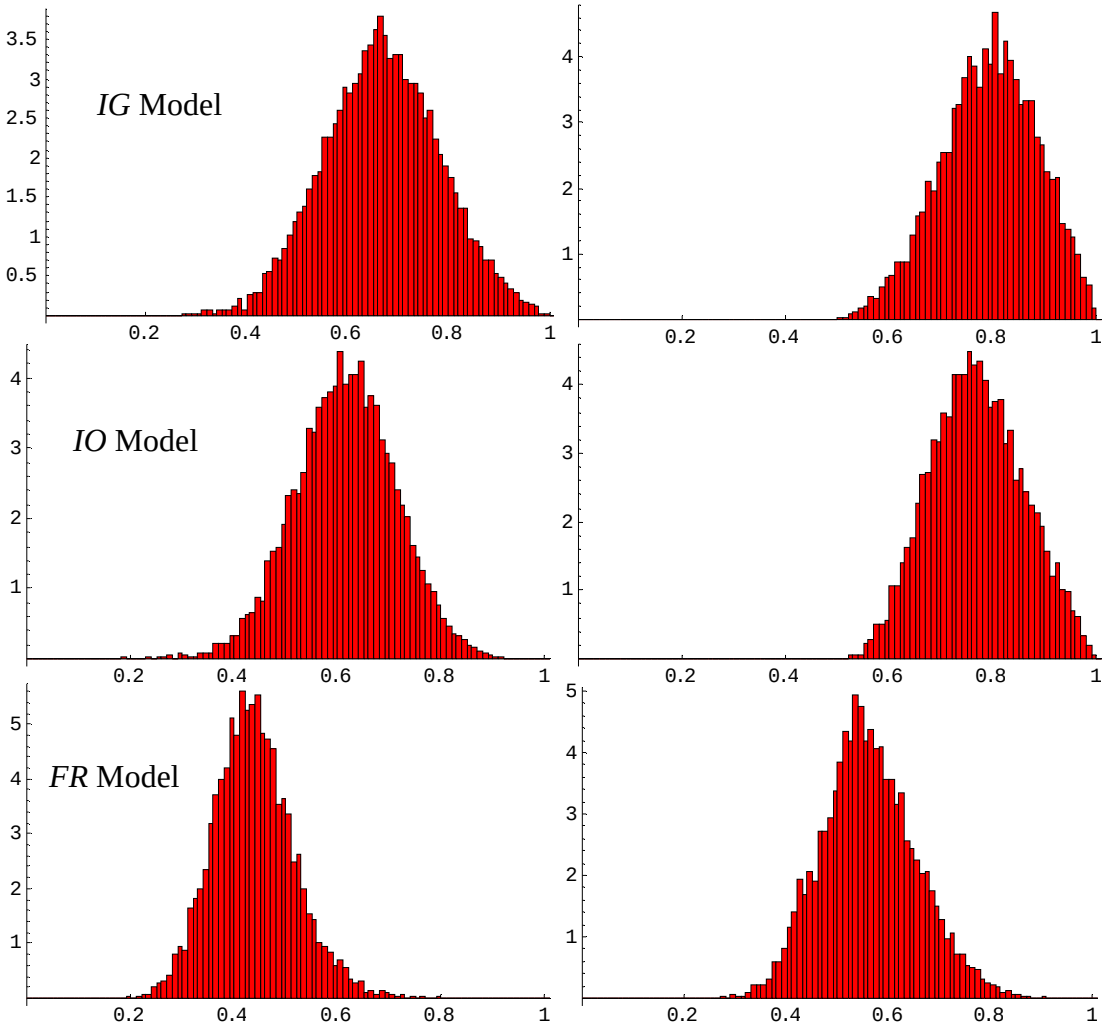
We first note the wider dispersion of rates of convergence under authoritarian regimes. Here the rate of convergence depends on two main factors: First, how fast is intergovernmental convergence and how much regard do citizens have for their governments. In contrast, under liberal governments there is one main factor determining the rate of convergence: the rate at which citizens converge. Second, *for a given political structure, the authoritarian arrangement converges faster than the liberal arrangement.*[24]

[24] This is confirmed by the KS-tests at higher than 0.999 significance level.

The impact of political structures is more ambiguous; comparing *IG* against *IO* models does not yield a clear picture; the test of *IG* against the *IO* models under the authoritarian arrangement rejects the hypothesis of equal distribution against the alternative of unequal distribution and the hypothesis of equal distribution against the alternative of slower *IG* at a significance level of 0.995. The test is unable to reject the hypothesis of equal distribution against the alternative of slower *IO* at the 0.9 significance level. The structural tests are thus only partially successful.

Popular and Unpopular Governments. We now consider a world of popular governments where citizens give more regard to their government than they give to anyone else. In the unpopular governments arrangement citizens give less regard to their governments than they give to anyone else.

Figure 5. Popular vs. Unpopular Governments



In all three models have popular governments faster rates of convergence than unpopular governments; this is confirmed by the KS tests at higher than 0.999

significance level. Under each of the popular and unpopular government arrangements the structural tests are all successful at higher than 0.999 significance level.

The Liberal/Popular Combinations. We now consider all possible combinations of liberal vs. authoritarian and popular vs. unpopular arrangements across each model. As one might expect from the previous two simulations, *popular authoritarian arrangements are the fastest-converging*, while the unpopular liberal arrangements are the slowest. Now take the *unpopular liberal* arrangement as a point of reference, as it is the slowest converging model. Moving in the IG model towards the *unpopular authoritarian* arrangement increases the speed of convergence by more than a move towards the *popular liberal* government arrangement. But the exact opposite occurs for IO and FR models: the move towards popular liberal arrangements increases the speed of convergence by more than the move towards unpopular authoritarian arrangements. All of these comparisons are confirmed by KS tests at higher than 0.999 significance level. The structural ranking tests were successful across all arrangements at 0.999 significance level, except for the comparison between the *IG* and *IO* models in the unpopular authoritarian arrangement, which passed the test only at 0.95 significance. Hence, we may conclude that if the ideal normative discourse conditions postulated by deliberative democracy are reflected by popular liberal political systems, *a federal system with features of deliberative democracy guarantees the fasted convergence to consensus.*

Figure 6.1. Liberal/Popular Combinations in the *IG* Model

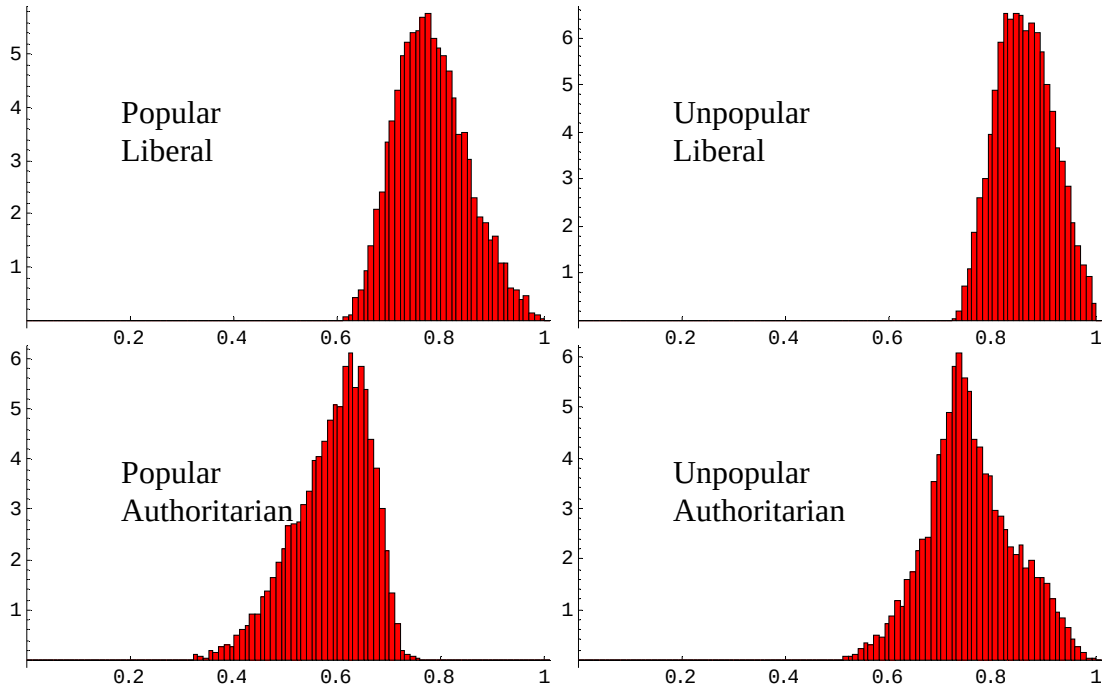


Figure 6.2 Liberal/Popular Combinations in the *IO* Model

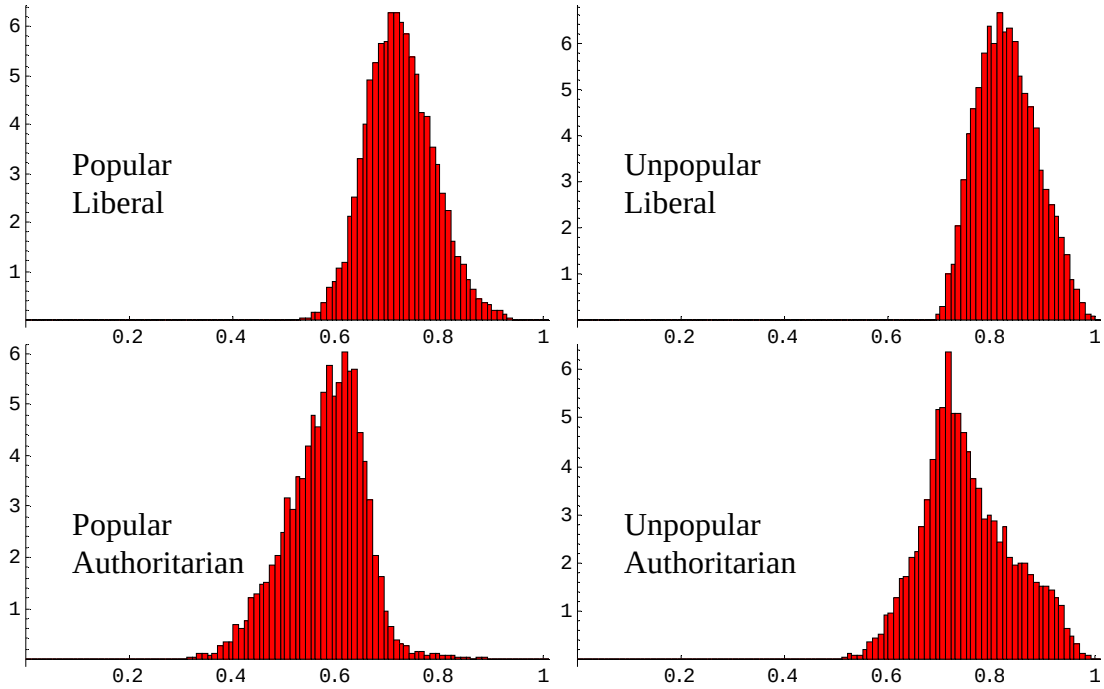
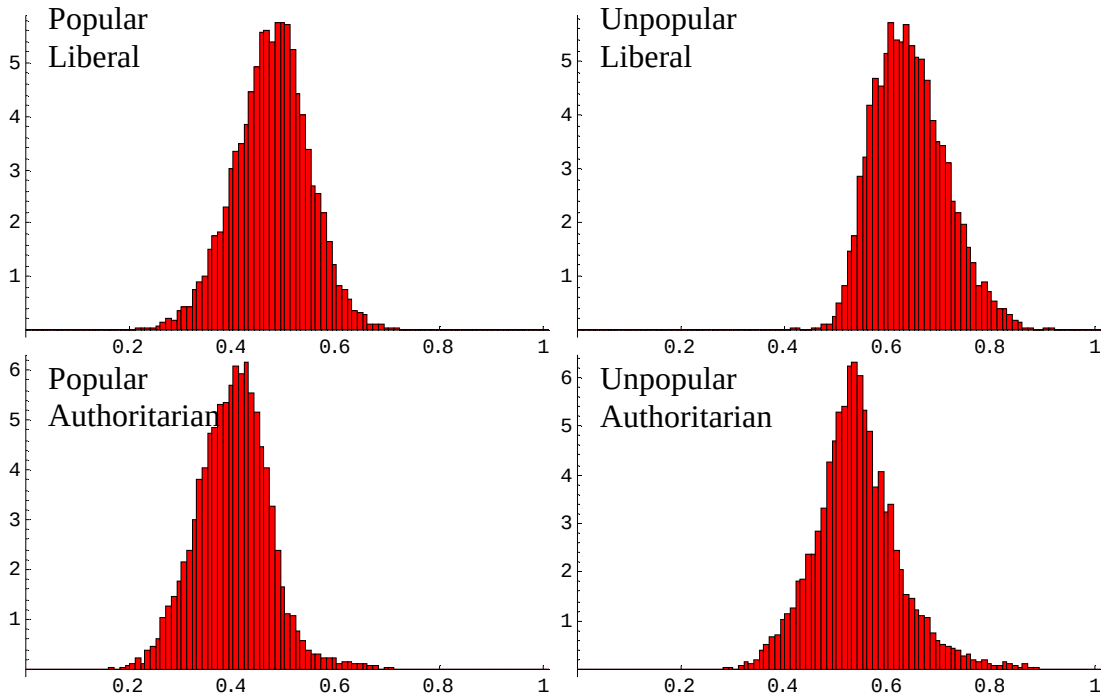
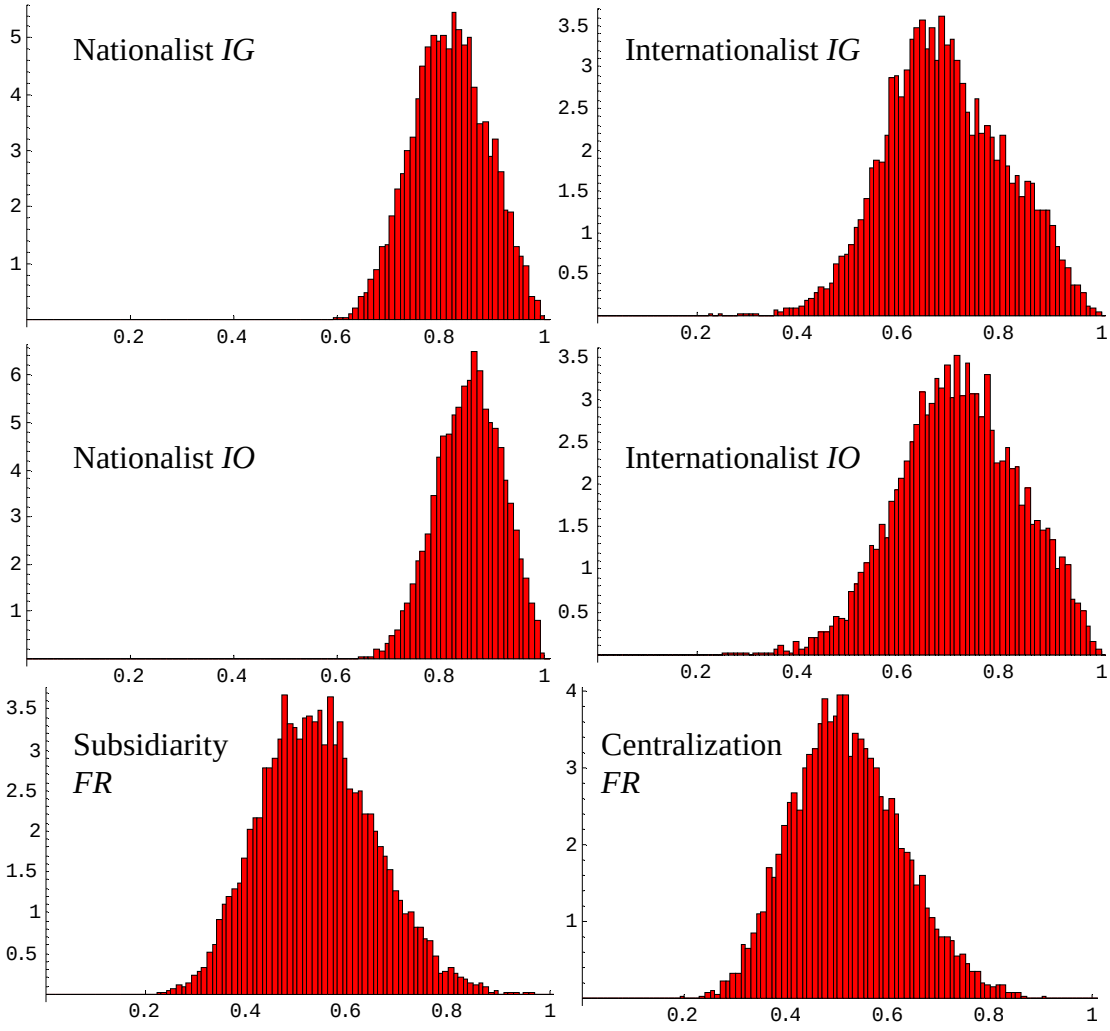


Figure 6.3 Liberal/Popular Combinations in the *FR* Model



Nationalism and Subsidiarity. We model *nationalism* in the *IG* and *IO* cases by simulating structures in which governments give less regard to international institutions than to domestic constituencies. The opposite is *internationalism*, where international links receive greater respect than domestic ones. Obviously, these categories do not apply to the Federal Republic. In the *FR* case, *subsidiarity* is the equivalent of nationalism when the Federal government gives high regard to the lower-level state governments, and *centralizing federalism* when links to local governments count less.

Figure 7. **Nationalism vs**. **Internationalism and Subsidiarity vs**. **Centralization**



Internationalist arrangements converge faster in both the IG and the IO cases and *centralization converges faster than the subsidiarity* arrangement; this is confirmed by the KS tests at higher than 0.999 significance. The structural ranking tests are not all successful. While the *FR* model is fastest across models, the *IG* beats the *IO* model in both the nationalist and the internationalist arrangements. This means that the institutions of federalism structure the flow of information and the process of political deliberation in such a way that political consensus on policy preferences among all citizens of the federation emerges most rapidly.

To summarize, the overall picture is that in general intergovernmental structures are likely to converge more slowly than the international organizations and the federal republic model converges the fastest of the three. Of course the nature of the political regime, its constitution and the standing of the federal government strongly affect the convergence to consensus and may disprove this statement in specific areas. But the important message of our simulations is: *centralising federalism is a device to overcome communitarianism.*

Conclusion

The theory of stochastic consensus presented here overcomes the paradox of simultaneously having to assume that political deliberation is depending on given normative conditions and at the same time creating them. From this new point of view, deliberation is not a normative postulate of democracy, but a consequence of bounded rationality. People deliberate because they seek actions coherent with their wills and wishes. Because they are limited in their cognitive capacities, they look at what they can learn from others and this leads them to adjust their preferences until they all see the world in the same way. Of course this concept of deliberation is unrelated to the idea of democracy. We have, however, shown that institutional structures of organizing the debate about political issues strongly influence the degree of dissent among citizens and the speed by which a society converges to a policy consensus. From that perspective, the institutional arrangements of deliberative democracy matter greatly.

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Annex: Matrices of political systems

Each parameter represents a transition probability in the weight matrix. Structural features of the stylized facts impose constraints in some of these values. For a fuller discussion of the model, see Collignon ans Al-Sadoon, 2006

The *IG* Model

$$\begin{bmatrix}
1-\alpha & 0 & \cdots & \cdots & 0 & & & & & & \alpha & 0 \\
0 & 1-\alpha & \cdots & \cdots & 0 & & & & & & \alpha & 0 \\
\vdots & \vdots & \ddots & & \vdots & & & \mathbf{0} & & & \vdots & \vdots \\
\vdots & \vdots & & \ddots & \vdots & & & & & & \vdots & \vdots \\
0 & 0 & \cdots & \cdots & 1-\alpha & & & & & & \alpha & 0 \\
 & & & & & 1-\beta & 0 & \cdots & \cdots & 0 & 0 & \beta \\
 & & & & & 0 & 1-\beta & \cdots & \cdots & 0 & 0 & \beta \\
 & & \mathbf{0} & & & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots \\
 & & & & & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots \\
 & & & & & 0 & 0 & \cdots & \cdots & 1-\beta & 0 & \beta \\
\delta/n & \delta/n & \cdots & \cdots & \delta/n & 0 & 0 & \cdots & \cdots & 0 & 1-\gamma-\delta & \gamma \\
0 & 0 & \cdots & \cdots & 0 & \sigma/m & \sigma/m & \cdots & \cdots & \sigma/m & \eta & 1-\eta-\sigma
\end{bmatrix}$$

The *IO* Model

$$\begin{bmatrix}
1-\alpha & 0 & \cdots & \cdots & 0 & & & & & & \alpha & 0 & 0 \\
0 & 1-\alpha & \cdots & \cdots & 0 & & & & & & \alpha & 0 & 0 \\
\vdots & \vdots & \ddots & & \vdots & & & \mathbf{0} & & & \vdots & \vdots & \vdots \\
\vdots & \vdots & & \ddots & \vdots & & & & & & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & \cdots & 1-\alpha & & & & & & \alpha & 0 & 0 \\
 & & & & & 1-\beta & 0 & \cdots & \cdots & 0 & 0 & \beta & 0 \\
 & & & & & 0 & 1-\beta & \cdots & \cdots & 0 & 0 & \beta & 0 \\
 & & \mathbf{0} & & & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots \\
 & & & & & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & \vdots \\
 & & & & & 0 & 0 & \cdots & \cdots & 1-\beta & 0 & \beta & 0 \\
\delta/n & \delta/n & \cdots & \cdots & \delta/n & 0 & 0 & \cdots & \cdots & 0 & 1-\gamma-\psi-\delta & \gamma & \psi \\
0 & 0 & \cdots & \cdots & 0 & \sigma/m & \sigma/m & \cdots & \cdots & \sigma/m & \eta & 1-\eta-\phi-\sigma & \phi \\
0 & 0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & \cdots & 0 & \mu & \nu & 1-\mu-\nu
\end{bmatrix}$$

The *FR* Model

$$\begin{bmatrix}
1-\alpha-\kappa & 0 & \cdots & \cdots & 0 & & & & & & \alpha & 0 & \kappa \\
0 & 1-\alpha-\kappa & \cdots & \cdots & 0 & & & & & & \alpha & 0 & \kappa \\
\vdots & \vdots & \ddots & & \vdots & & & \mathbf{0} & & & \vdots & \vdots & \vdots \\
\vdots & \vdots & & \ddots & \vdots & & & & & & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & \cdots & 1-\alpha-\kappa & & & & & & \alpha & 0 & \kappa \\
 & & & & & 1-\beta-\rho & 0 & \cdots & \cdots & 0 & 0 & \beta & \rho \\
 & & & & & 0 & 1-\beta-\rho & \cdots & \cdots & 0 & 0 & \beta & \rho \\
 & & \mathbf{0} & & & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots \\
 & & & & & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & \vdots \\
 & & & & & 0 & 0 & \cdots & \cdots & 1-\beta-\rho & 0 & \beta & \rho \\
\delta/n & \delta/n & \cdots & \cdots & \delta/n & 0 & 0 & \cdots & \cdots & 0 & 1-\gamma-\psi-\delta & \gamma & \psi \\
0 & 0 & \cdots & \cdots & 0 & \sigma/m & \sigma/m & \cdots & \cdots & \sigma/m & \eta & 1-\eta-\phi-\sigma & \phi \\
\frac{\theta}{n+m} & \frac{\theta}{n+m} & \cdots & \cdots & \frac{\theta}{n+m} & \frac{\theta}{n+m} & \frac{\theta}{n+m} & \cdots & \cdots & \frac{\theta}{n+m} & \mu & \nu & 1-\mu-\nu-\theta
\end{bmatrix}$$